Balance of payments and international parity conditions between Germany and USA under fixed and floating exchange rates

While in the standard neoclassical macroeconomic model and more in general in a real world-economy savings and investments must equalize, in an open economy the fact that savings and investments of a nation equalize at all times may represent more the exception rather than the rule. Since the current account must be equal to savings minus investments, the issue about non stationary international parities put forward by Rogoff (1996), Cumby and Obstfeld (1981)[1] may eventually find a response in the intertemporal saving and investment decisions.

Nations borrow or lend money from each other. Financial capitals flow among countries and, although in the long run an equalization between savings and investments should be attained at country level, this equalization is not secured at all times and may require many years once it has occurred to occur again. Viewing at the current account, the financial and capital account in the balance of payments of a nation, it is very unlikely that both the current account and the financial and capital account zero. The balance of payment may (must in a clear floating)

[1]See Juselius and MacDonald 2000.

zero, the current account and the financial and capital account may balance each other but it is rare that for instance the current account zeroes. This fact may be crucial in understanding the reason behind the non stationarity of international parity conditions, namely the *ppp* and the *uip*, and their stationary interaction.

MacDonald (2000) in our opinion offered one possible and sensible reason for the robustness of a long-run relationship between *ppp* and *uip* found by Juselius (1995) and further validated by Juselius and MacDonald (2000 and 2003). The explanation about the dependability of the cointegration relation between *ppp* and *uip* stems from a simple balance of payment model which is based on a double entry accounting relation, the balance of payments, and few assumptions around the accounts of the balance of payments and international parity relationships. In line with the methodology suggested by Juselius (1995 and 2005) this paper essentially tests some basic relationships suggested in the balance of payment model by MacDonald (2000) and a similar cointegration relationship between the *ppp* and the *uip* between Germany and USA for both the Post Bretton-Woods and the Bretton-Woods periods will be found, although the two epochs were characterized by distinct exchange rate regimes.

Section 1 presents a stylized theoretical background relating the balance of payments account with international parity conditions suggesting that international parity conditions are likely to be linearly interrelated. Section 2 shows that a statistical cointegrated VAR model and its VMA representation are derived from the stylized theoretical model once *ppp* and *uip* are assumed to follow random walks. Section 3 presents a general version of the statistical model that is likely to approximate well the data since it takes into account of the historical events and the context in which the data were generated. Subsections 3.1 and 3.2 present a $I(1)$ analysis consistent with the methodology described in Juselius (2005) that tests some long-run relationships suggested in the theoretical model for both the Post Bretton Woods and the Bretton Woods periods. Section 4 concludes showing that the theoretical model is essentially valid for both periods and that the relationships between parity conditions in a pegged exchange rate regime seem to be a particular case of the relationships found for floating exchange rates

1 Theoretical background: a balance of payments model

The balance of payments provides key information on the international relationships of a country. It should measure its trade and financial flows with the rest of the world. The balance of payments of a country is made of the current account balance, the capital and financial account balance, net errors omissions and the change in the foreign exchange reserves. Because of the double entry accounting the following balance of payments identity must always hold:

$$CA + KA + \varepsilon \equiv \Delta FXR \tag{1}$$

Where, CA is the balance on current account, KA is the capital and financial account, ΔFXR is the change of foreign exchange reserves and ε stands for net errors and omissions.[2]

The historical evidence from Germany and the USA shows that both the current and financial accounts imbalances can last for several years although they eventually zero after decades and this ultimately hinges on the intertemporal choices about the savings and the investments in a nation. In fact from the national accounts the current account must be equal to the spread between savings and investments:

$$CA \equiv -KA + \Delta FXR - \varepsilon \equiv S - I \tag{2}$$

where $-KA + \Delta FXR - \varepsilon$ represents the net liabilities of the home country with respect to the rest of the world that in turn are always equal to the spread between savings S and investments I. Any country running a deficit (surplus) in the current account has either to borrow (lend) foreign currency or reduce (increase) the official foreign exchange reserves. Thus, given the fact that the official foreign exchange reserves are a finite amount and the financing through the capital account by the rest of the world the current account has to converge in the very long run towards parity, the current account may balance very slowly.

[2]In a pure clean floating central banks do not buy or sell currencies and gold, then changes in official reserves cannot subsist. The change in the foreign exchange reserves ΔFXR should fade away in an approximate clean floating exchange rate regime.

1.1 The Current Account, the Trade Balance and *ppp*

The current account may be further decomposed in its components. The current account is defined as the sum of the trade balance or net exports NX_t (value of exports EXP_t minus value of imports in home currency $S_t IMP_t$ where S_t is the spot exchange rate) and other elements such as net interest payments on net foreign assets $i_t^{'} nfa_t$:

$$CA_t \equiv NX_t + i_t^{'} nfa_t \tag{3}$$

where CA_t and NX_t are valued at current prices at time t.

The trade balance NX_t is in equilibrium when $\ln EXP_t - \ln IMP_t - s_t = 0$ where s_t denotes the log of the spot exchange rate at time t. There is a net trade surplus when $\ln EXP_t - \ln IMP_t - s_t > 0$ and a trade deficit whenever $\ln EXP_t - \ln IMP_t - s_t < 0$.

Assuming that the value of net exports of a country depends on its price competitiveness and the relative domestic income:

$$NX_t = f\left(-p_t + p_t^* + s_t, y_t, y_t^*\right) \tag{4}$$

with f indicates generic function, p_t the log of the domestic price p_t^* the log of the foreign price, y_t and y_t^* the log of domestic and foreign incomes respectively. If home (foreign) price index grows, ceteris paribus, the value of home (foreign) net trade falls the Marshall-Lerner condition is implied in the last equation. If the last equation holds, an equilibrium in the trade balance exists when $-p_t - p_t^* - s_t = 0$, i.e. when the *ppp* holds. The trade balance would move to a surplus or a deficit whenever the *ppp* is negative or positive respectively. The trade balance is certainly the main term of the current account and the overall balance of the current account may be expected to follow the movements of the trade balance:

$$CA_t = f\left(-p_t + p_t^* + s_t, y_t, y_t^*\right) + i_t^{'} nfa_t \tag{5}$$

If f can be approximated by a linear function:

$$CA_t = \delta_1\left(-p_t + p_t^* + s_t\right) + \delta_2 y_t, + \delta_3 y_t^* + i_t^{'} nfa_t \tag{6}$$

with $\delta_1, \delta_2, \delta_3 > 0$. The parameter δ_1 can be interpreted as the responsiveness of the current account to *ppp* imbalances. A small (high) value of the parameter δ_1 implies a small (high) responsiveness of net trade to the price differential between the two countries. If $\delta_1 \to \infty$, the last equation reduces to the condition of purchasing power parity (MacDonald 2000) as implied in most neoclassical macroeconomic models.

1.2 The Financial and Capital Account and *uip*

The financial and capital account represents the net financial inflows and it may include the change in official foreign exchange reserves. Denoting $S_t V_t^*$ the value of financial investment of home residents abroad and V_t is the value of investments of foreign residents in the home country when the capital account is in equilibrium $\ln V_t - \ln V_t^* - s_t = 0$. If $\ln V_t - \ln V_t^* - s_t > 0$ or $\ln V_t - \ln V_t^* - s_t < 0$ the capital account will be respectively in surplus or in deficit. Differencing $\Delta \ln V_t - \Delta \ln V_t^* - \Delta s_t$ will be zero in case of a balanced capital account, will be positive in case of surplus and negative in case of deficit. The difference of a log represents a growth rate and the growth rate for the value of capital is the interest rate. Thus, $\Delta \ln V_t - \Delta \ln V_t^* - \Delta s_t \equiv i_t - i_t^* - \Delta s$. If $i_t - i_t^* - \Delta s = 0$ the capital account should balance as there would no more incentive to move financial capitals. If $i_t - i_t^* - \Delta s > 0$ there would be a capital account surplus (the growth rate of the value of domestic investments exceeds the one of foreign investments) and a deficit when $i_t - i_t^* - \Delta s < 0$

Assuming rational agents, $\Delta s = E_t \Delta_l s_{t+l} + v_t$ with v_t *i.i.d.*, an equation that relates the capital account with the *uip* may be also assumed

$$KA_t = -g\left(-i_t + i_t^* + E_t \Delta_l s_{t+l}\right) \tag{7}$$

where g denotes a generic function and $(-i_t + i_t^* + E_t \Delta_l s_{t+l})$ is the definition of *uip*. If g can be approximated by a linear function:

$$KA_t = -\mu\left(-i_t + i_t^* + E_t \Delta_l s_{t+l}\right) \tag{8}$$

where $\mu > 0$, i_t^l denotes an interest rate yield, E_t denotes the conditional expectations operator on the basis of time-t information set.

If $\mu \to \infty$, the last equation reduces to the condition of uncovered interest rate parity (MacDonald 2000). The parameter μ can be interpreted as the responsiveness of the capital movements that enter in the capital and financial account to *uip*. A small (high) value of the parameter μ implies a small (high) responsiveness of capital movements to the net interest rate differential.

1.3 Combining the Current and the Capital Accounts

If KA_t is measured including both the change in the foreign exchange reserves and the statistical discrepancy, from the definition of balance of payment

$$CA_t \equiv -KA_t \tag{9}$$

for any exchange rate regime.

Hence:

$$-\delta_1 \left(p_t - p_t^* - s_t\right) + \delta_2 y_t, + \delta_3 y_t^* + i_t' nfa_t = \mu \left(-i_t + i_t^* + E_t \Delta_l s_{t+l}\right) \tag{10}$$

which is a relation which assumes proportionality between *ppp* and *uip*:

$$-\omega ppp = uip - \left(\frac{\delta_2}{\mu} y_t + \frac{\delta_3}{\mu} y_t^* + \frac{1}{\mu} i_t' nfa_t\right) \tag{11}$$

with $\omega = \frac{\delta_1}{\mu}$ and $\mu \neq 0$. The parameter ω can be interpreted as the responsiveness of the capital movements that enter in the capital and financial account to *uip* compared to the responsiveness of the current account that to *ppp* imbalances . A high (small) value of the parameter ω implies a smaller (high) responsiveness of capital movements to the net interest rate differential than the responsiveness of the current account to *ppp* imbalances.

The Bretton-Woods period witnessed a gradual removal of most of the trade restrictions that characterized the foregoing period. The close to free trade environment in goods and services promoted economic growth, but short term capital flows were seen as a source of disturbance to exchange rate stability and a menace undermining an enshrined pillar for stable growth.

Due to the restrictions and the heavy regulation for capital movements typical of the Bretton-Woods period, we expect a slower responsiveness of capital movements to the net interest rate differential than the current account to *ppp* imbalances. Thus it may be expected a higher value of the parameter ω for the Bretton-Woods period than the recent floating exchange rate experience.

Setting $l = 1$, in accordance with the vast empirical evidence supporting the relative purchasing power parity if speculators form exchange rate expectations on the basis of inflationary prediction, $E_t\Delta s_{t+1} = E_t\Delta p_{t+1} - E_t\Delta p^*_{t+1}$, we have:

$$-\omega\left(p_t - p_t^* - s_t\right) = \left(-i_t + i_t^* + E_t\Delta p_{t+1} - E_t\Delta p^*_{t+1}\right) - \left(\frac{\delta_2}{\mu}y_t + \frac{\delta_3}{\mu}y_t^* + \frac{1}{\mu}i_t^{'}nfa_t\right) \qquad (12)$$

Either the inflation rate is non stationary or stationary, rational expectations in prices may be modeled by $(\Delta p_t - \Delta p_t^*) = E_t(\Delta p_{t+1} - \Delta p^*_{t+1}) + v_t$ with v_t *i.i.d.*.

Thus:

$$-\omega\left(p_t - p_t^* - s_t\right) = \left(-i_t + i_t^* + \Delta p_t - \Delta p_t^*\right) - \left(\frac{\delta_2}{\mu}y_t + \frac{\delta_3}{\mu}y_t^* + \frac{1}{\mu}i_t^{'}nfa_t\right) + v_t \qquad (13)$$

which is exactly the testable equation which, as we shall see, holds for both the Bretton-Woods and the Post Bretton-Woods period. Equation (13) holds in the case both *ppp* and *uip* are unit root non stationary $I(1)$ but cointegrate. The real term $\left(\frac{\delta_2}{\mu}y_t, +\frac{\delta_3}{\mu}y_t^*\right)$ jointly with $\frac{1}{\mu}i_t^{'}nfa_t$ may be either $I(0)$ or $I(1)$ increasing the cointegration rank. In the next section central to our analysis will be to check whether the *ppp* and *uip* cointegrate notwithstanding a radical change in the exchange rate regime: the passage from the pegged exchange rate system and capital restrictions of the Bretton-Woods period to the recent floating exchange rate system with fairly free capital movements. Relative income effects and net interest payments on net foreign assets will not be analyzed and will be dropped in the statistical analysis, although the same analysis could be potentially extended to include them.

The last equation may be also interpreted as a general representation of an equilibrium exchange rate similar to equation (6) in MacDonald (2000)

$$s_t = p_t - p_t^* + \frac{1}{\omega}\left(-i_t + i_t^* + \Delta p_t - \Delta p_t^*\right) - \frac{1}{\omega}\left(\frac{\delta_2}{\mu} y_t + \frac{\delta_3}{\mu} y_t^* + \frac{1}{\mu} i_t^{'} nfa_t\right) \tag{14}$$

which shows that higher domestic interest rates, ceteris paribus, brings about an appreciation of the domestic currency consistent with overwhelming empirical evidence, while higher domestic inflation is related with a depreciation of the domestic currency. When the exchange rate is fixed, this still remains determined by the same supply and demand determinants of the flexible exchange rate regime. Thus the equation for an equilibrium exchange rate should remain invariant of the exchange rate regime.

Although we will not go much further in the analysis we would like to point out that these equations are related to important macroeconomic aggregates.

The current account depends on saving and investment decisions; CA in fact must be equal to the spread between savings S and investments I. The equilibrium exchange rate equation may be further rewritten as:

$$s_t = p_t - p_t^* - \frac{1}{\delta_1}(S - I) + \frac{1}{\delta_1}\left(\delta_2 y_t + \delta_3 y_t^* + i_t^{'} nfa_t\right) \tag{15}$$

which shows that when domestic savings exceed investments the domestic currency should, ceteris paribus, appreciate.

Again, if KA_t is measured including both the change in the foreign exchange reserves and the statistical discrepancy, the following equation shows that the domestic interest rate fall (rise) whenever domestic savings exceed (are lower than) domestic investments:

$$i_t = i_t^* + \Delta p_t - \Delta p_t^* - \frac{1}{\mu}(S - I) + \frac{1}{\mu}\left(\delta_2 y_t + \delta_3 y_t^* + i_t^{'} nfa_t\right) \tag{16}$$

The last two equations also show that a necessary condition for the *ppp* and *uip* to hold is $S = I$, condition that represents an exception rather than the rule in an open economy. The reason for $S \neq I$ resides in the intertemporal saving and investment decisions and may depend on a variety of factors such as expected growth, risk of investments, political stability etc. we do not further investigate in this work.

2 A link between a stochastic balance of payment model, unit roots and cointegration

It is maintained that both the *ppp* (Rogoff 1996) and *uip* (Cumby and Obstfeld 1981) changes occur *stochastically* as a random walk as if:.

$$ppp_t = ppp_0 + \sum_{i=0}^{t} \varepsilon_i \tag{17}$$

$$uip_t = uip_0 + \sum_{j=0}^{t} \varepsilon_j \tag{18}$$

where $\sum_{i=0}^{t} \varepsilon_i$ and $\sum_{j=0}^{t} \varepsilon_j$ represent permanent shifts of ppp_t. and uip_t respectively. $\sum_{i=0}^{t} \varepsilon_i$ and $\sum_{j=0}^{t} \varepsilon_j$ represent also the non stationary components of ppp_t. and uip_t respectively. If a linear relationship between ppp_t and uip_t is maintained as it may be implied by the balance of payment model by MacDonald (2000) $\sum_{i=0}^{t} \varepsilon_i = \frac{1}{\omega} \sum_{i=0}^{t} \varepsilon_j$ where $\frac{1}{\omega}$ is a constant. If a linear relationship between ppp_t and uip_t exist then the non stationary components in ppp_t. and uip_t cancel out and only the stationary part ppp_0 and uip_0 remain. In this case we say that ppp_t and uip_t share a common stochastic trend $\sum_{i=0}^{t} \varepsilon_i$.

In fact if $uip_t = -\omega ppp_t$:

$$uip_0 + \omega ppp_0 = -\sum_{j=0}^{t} \varepsilon_j - \omega \sum_{i=0}^{t} \varepsilon_i \tag{19}$$

where $uip_0 + \omega ppp_0$ are stationary component while components in the r.h.s. represent the non stationary components in ppp_t. and uip_t that produce a stationary relation, i.e. they are cointegrated. In this sense, ppp_t and uip_t like most macroeconomic variables follow a *stochastic* trend $\sum_{i=0}^{t} \varepsilon_i$ or $\sum_{i=0}^{t} \varepsilon_j$ which is *common* as (neglecting stationary components) $\sum_{i=0}^{t} \varepsilon_i = -\frac{1}{\omega} \sum_{i=0}^{t} \varepsilon_j$. The variable related to the common trend is evidently the *driving force* of the system, which pushes the system away from steady state, generating a non stationary behavior in the variables.

What is implicit in the stochastic version of the balance of payment model is that the system is essentially stable. In fact, we removed random innovations from time series we would have a stationary stable system that reproduces itself. However, without random innovations, in a linear setup like this the only solution that could actually last for ever is when both parities and the balance accounts zero. The reason is that a current account imbalance has to be financed from abroad and this debt must be repaid in the future. Conversely, the inclusion of random innovations allow for imbalances in the balance of payments accounts leaving open the possibility to honor current account deficits in the future.

Equations (17) and (18) can be summarized with the following *moving average representation*, VMA:

$$x_t = C \sum_{i=0}^{t} \varepsilon_i + x_0 \tag{20}$$

where $x_t' = [uip_t, ppp_t]$, $C = \beta_{\perp}\alpha_{\perp}^{'}$, $\alpha_{\perp}^{'} \sum_{i=0}^{t} \varepsilon_i$ the common stochastic trend, $\beta_{\perp}^{'} = [1, -\omega]$, $\alpha_{\perp}^{'} = [1, 0]$.

The vector moving average representation VMA just described is equivalent to the vector autoregressive model VAR formulation of the same $I(1)$ model. While the VMA representation is useful for the analysis of the common trends that have generated the data, the VAR model enables us to single out the long run relations in the data. Inverting this moving average form under suitable conditions we obtain the following stationary very simple *vector autoregressive* (VAR) model (Granger representation theorem, see Engle and Granger 1987, Johansen 1995):

$$\Delta x_t = \alpha\beta' x_{t-1} + \varepsilon_t \tag{21}$$

where:

- A vector orthogonal to $\beta_{\perp}^{'}$ is given by $\beta' = \begin{bmatrix} 1 & \omega \end{bmatrix}$.
- x_t is a cointegrated process as the cointegrated relation $\beta_1' x_{t-1} = uip_{t-1} + \omega ppp_{t-1}$ implies a stationary steady-state relationship among the levels of the variables belonging to the vector process x_{t-1}.
- $\alpha' = \begin{bmatrix} 0 & 1 \end{bmatrix}$ defines the direction and its length the speed of adjustment, the *pulling forces* (see first quadrant of Fig. 1), which pull the process towards the steady state

relationship and the *attractor set* $\beta_\perp$ (the diagonal lines in the first quadrant of Fig. 1) whenever a shock hits the system[3]. Note that the unit vector in $\alpha'_\perp$ corresponds to a zero row in α which is a condition for *weak exogeneity* in this case in *uip*, i.e. a shock in *uip* would have a long run impact on the other variables of the system without being affected by them. Of course we might have modeled the weak exogeneity for *ppp* a similar way.

- $\Pi = \alpha\beta'$ is a matrix with rank 1, the number of basis vector which span the cointegration vector space (in this case a line).

Cointegration is a concept that has been developed by Hendry (1986), Engle and Granger (1987) and Johansen (1988). The idea is that although economic variable may move in a non stationary way, linear combinations of these variables are characterized by a lower degree of integration than the vector process x_t and may persist unchanged over a period of time. They pointed out that a linear combination of two or more non stationary series, which for example contain unit roots, hence they are $I(1)$, may be stationary. If a stationary $I(0)$ linear combination exists, the non stationary time series are cointegrated. The stationary linear combination is called cointegrating equation and may be interpreted as a long-run equilibrium (steady-state) relationship. In the stylized *vector autoregressive* (VAR) model just described we have one cointegration relations ($\beta' x_{t-1} = uip_{t-1} + \omega ppp_t$) and one pushing force (uip_t) which would be allegedly the source of the stochastic trend in the system. The rank of the Π matrix is equal to the number of stationary relations between the levels of the variables, i.e. the number of long run steady states towards which the process starts adjusting when it has been pushed away from the equilibrium (Hansen and Juselius 2000).

The C matrix is informative regarding the total effects of the stochastic driving *pushing force* of the system which pushes the system along the attractor set $\beta_\perp$. The rank of the C matrix is equal to the number of stochastic trends that push economic variables away from steady states. In absence of stochastic trends the system would never change (see the arrow lines in Fig. 1), and the occurrence of exogenous shocks, for instance in the interest rate is necessary to move the process along the attractor set. A column of C shows the effect of a cumulated shock on each variable of the system. Therefore, the one column of zeroes in the

[3]See also Juselius 2005, Chapter 5, on which this Section is based.

C matrix implies that ppp_t would not permanently affect any variable of the system. This can also be seen in the α matrix: there is one unit column vectors in α, which corresponds to ppp_t implying that ppp_t is exclusively adjusting to the long-run cointegration relation and its unanticipated shocks would have no permanent effect on the variable of the system.

Therefore each shock in interest rates would affect permanently ppp_t, as the first column of the C matrix have all non zero coefficients, i.e. the variables in the system are driven by the common stochastic trend defined as the cumulated shocks in uip_t. The rows of the C matrix shows the weight with which each of the variables have been affected by cumulated shocks and the one unit row vector in C means that uip is a common trend in this stylized model. uip is not affected by any other variable in the system and thus would be called exogenous, or *strongly exogenous*.

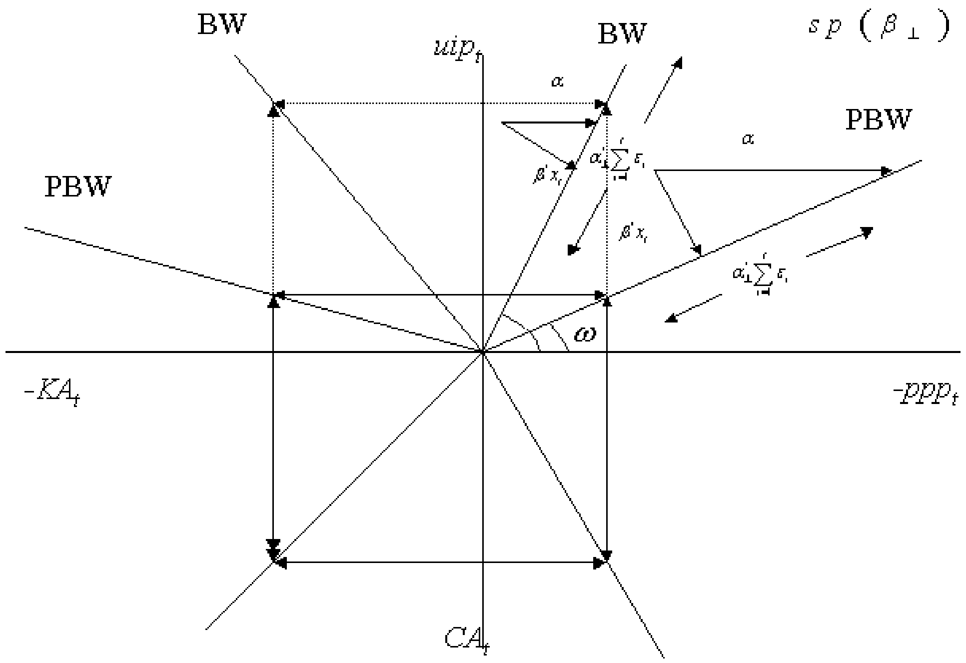

Fig. 1: The process $x_t' = [uip_t, ppp_t]$ is pushed along the attractor set by the common trends and pulled towards the attractor set by the adjustment coefficients.

3 Statistical analysis: the I(1) model

The model just described is a very stylized model. Although this model might allow common features in macroeconomic time series such as unit root non stationarity and cointegration, it also implies a lot of restrictions and hypothesis that are normally not adequate to describe even approximately the data. In fact the underlying crucial assumption of this model, independence of errors and parameter constancy, would hardly be satisfied by this very restricted model. This model assumes that there is no short-run dynamics, no transitory, intervention and shift dummies are modeled implying no outliers and no regime shift and shifts in the equilibrium means, hence ignoring the historical events and context in which the data were generated. However when analyzing macroeconomic data it often matters to allow for policy intervention as well as structural breaks (Johansen *et al.* 2000) and shifts in the equilibrium means so that a more general version of the vector autoregressive model that approximate well the data also in terms of independence of disturbances and parameter stability is needed.

A general form of the $I(1)$ VAR model formulated in the error correction form is

$$\Delta x_t = \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \Pi x_{t-1} + \mu_0 + \mu_1 t + \Psi_0 Dp_t + \Psi_1 Dtr_t + \Psi_2 Dq_t + \Psi_3 DS_t + \varepsilon_t \qquad (22)$$

with $\varepsilon_t \sim N_p(0, \Sigma)$, $i = 1...n$, μ_0 and μ_1 constants, $t = 1, ..., T$ where p is the dimension of the VAR model, $x_t' \sim I(1)$, k is the lag length, DS_t is a vector of mean shift dummy variables which accounts for a mean in Δx_t and cumulates to a broken trend in x_t serving to capture regime shifts, Dp_t a vector of deterministic components with permanent effect such as intervention dummies, Dtr_t a vector of transitory shock dummy variables, Dq_t centered seasonal dummies which sum to zero in samples comprising complete years, Γ_1,..., Γ_{k-1}, Ψ matrices of freely varying parameters and $\Pi = \alpha\beta'$ where α and β are $p \times r$ matrices of full rank, r is the rank of the Π matrix, and $\beta' x_t$ is stationary, i.e. the stationary relations among non stationary variables. A constant, a trend and shift dummies can be restricted to lie in the cointegration space. In this general form, the constant, time trend and the shift dummies are thus decomposed into two new vectors one of which lies in the cointegration space. This general model may be always restricted and some terms may be dropped if for instance economic theory suggests so then again it takes into account of transitory shocks, permanent interventions and regime shifts grounded on historical facts and we may eventually obtain the stylized model in summarized in Section (2). In this extended model a variable to be strongly exogenous needs that the corresponding row in the α matrix zeroes as well as the corresponding row in the short-run matrices Γ. If this is the case for that variable there is neither a long run nor a short run effect from other variables in the system. Conversely if the corresponding row in the short-run matrices Γ has significant values, there is a short run effect from other variables of the system and if the corresponding row in the α matrix zeroes, there is no long run effect from the other variables of the system. In this case the variable is called *weakly exogenous.*

The corresponding VMA representation of the general for of the $I(1)$ VAR model is:

$$\begin{aligned} x_t &= C\sum_{i=1}^{t}\varepsilon_i + C\sum_{i=1}^{t}\Psi_0 Dp_i + C\sum_{i=1}^{t}\Psi_1 Dtr_i + C\sum_{i=1}^{t}\Psi_3 DS_i + \\ &\quad + C^*(L)\left(\varepsilon_t + \Psi_0 Dp_t + \Psi_1 Dtr_t + \Psi_3 DS_t + \mu_0 + \mu_1\right) + X_0 \end{aligned} \tag{23}$$

where $C = \beta_\perp \left(\alpha'_\perp \left(I - \sum_{i=1}^{k-1}\Gamma_i\right)\beta_\perp\right)^{-1}\alpha'_\perp$, $\alpha_\perp$ and $\beta_\perp$ are $(p-r)\times(p-r)$ matrices orthogonal to α and β, C matrix is of reduced rank of order $(p-r)$ and X_0 the initial values. $C^*(L)$ is an infinite polynomial in the lag operator L. The component $\alpha'_\perp \sum_{i=1}^{t}\varepsilon_i$ represents common stochastic trends of the process, the $C\sum_{i=1}^{t}\Psi_3 DS_i$ captures a broken trend in x_t while $C\sum_{i=1}^{t}\Psi_0 Dp_i$ and $C\sum_{i=1}^{t}\Psi_1 Dtr_i$ are a shift in the level of x_t and a temporary change in x_t respectively.

3.1 Choice of the variables and data set

The variables that enter in equation (14) are, the home price index p_t, the foreign price index p_t^*, the home interest rate i_t, the foreign interest rate i^*, the spot exchange rate s. In this analysis we do not investigate the effect of real incomes and foreign debt, although these variables can be very important in the analysis of the system. The overall effect of these variable could likely be to include a stationary component or a non stationary component cointegrating with ppp and uip. As a partial justification for this shortfall, let us say that reducing at minimum the number of variables often helps in identifying the cointegration relations and cointegration relations remain valid in a more extended model according to the known 'invariance' property of cointegration relations in extended sets. If cointegration is found within a small set of variables, the same cointegration relations should be valid within any larger set of variables.

We decided to start the analysis of our system with 5 variables including the long bond interest rates, i_t^l and i_t^{l*}, but excluding short term interest rates i_t^s, i_t^{s*}. Then we analyzed a system with 7 variables, $x_t' = \left[\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, i_t^s, i_t^{s*}, ppp_t\right]$, including short term interest rates. Since results are perfectly consistent with each other and very similar we will show the results of the analysis which includes short term interest rates for three reasons. First, in a relation such as equation (14) interest rates are included but it is not specificied if they should be short or long term interest rates. Second, short term interest rates are more directly linked with the monetary policy of the central banks, rather than long term interest rates, which more likely depend on other variables such as expectation of economic growth etc. Third, including both rated we should be able to understand which interest rates were the driving forces of the system and see whether short term interest rates affect long term interest rates or the other way round.[4]

The focus of this paper is on two countries, Germany and USA, a subset of the post Bretton-Woods (1975-1998) and a subset of the Bretton-Woods period (1959-1969) will be analyzed in the next two sections respectively. The decision to show first the analysis for the post Bretton Woods period is not by chance: the results of the analysis of the Bretton-Woods era, of the pegged exchange rate regime, are consistent with the results of the analysis for the post Bretton-Woods period. Moreover it does seem to us that what happened during the pegged exchange rate regime could be a theoretical and particular case of what could happen during the floating exchange rate regime.

The choice of the countries and the division of the sample period may be so justified:

[4]Treasury Bill rates are more closely linked to the monetary policy than long term interest rates as bond rates with a maturity of ten years. In fact, given its monopoly over the creation of base money, the central bank can fully determine the official interest rate and exert a dominant influence on money market conditions steering money market interest rates having an impact on short term interest rates (ECB 2004). Conversely, the impact of money market rate changes on interest rates at long maturities (e.g. government bond yields) is less direct as these rates depend to a large extent on market expectations for long term growth and inflation trends (ECB 2004). In general, changes in the central bank's official rates do not normally affect long term rates unless they lead to a change in market expectations on long term economic trends (ECB 2004). Including short term interest rates, we can test whether short term interest rates shocks normally do not lead to changes in long term interest rates as the ECB maintains unlike the standard expectations model of the term structure for which short rates drive long rates.

- Macroeconomic relations may change when the structure of the economy or policy regimes change as pointed out in the *Lucas critique*. Therefore it may be worth to divide the sample in regime periods, and conduct one specific analysis for the post war era and another for the post Bretton-Woods period. It should help to prevent parameter instability which may be caused by structural changes.

- The analysis limited to the period December 1959- August 1969 is before the Bretton-Woods collapse and some years after the rounding off of the WWII post-war recovery and the end of the Korean war in 1953. The period was characterized by a rather successful operation of a pegged exchange rate regime, although the system was slowly but steadily going into crisis already in the last years of 1960s when the US involvement in the Vietnam war became markedly priced for the deploying of a massive military force in 1968, which created unsustainable imbalances in payments culminated in August 1971 with the resolution by USA to suspend the convertibility of the US dollar in gold on which the Bretton-Woods system was based. The ending date in August 1969 was also chosen because the Germany authority in Sept. 1969 decided the German mark to freely float in response to speculative attacks in the currency market and too many dummies would have otherwise necessary to take into account of all these historical facts. The start date of July 1975 is rather convenient as it leaves out the most turbulent years of the collapse of the Bretton-Woods system and the oil crisis that would have lead us to include too many dummies in our linear analysis. The end date of January 1998 is instead before the introduction of the Euro.

- The two countries, Germany and USA, are certainly two 'big' countries if considered during the last thirty years. In the last 25-30 years, a change in policy in one of the two countries would have probably affected the other country. However in the immediate post war and the following period 1959-1969 the financial hegemony of USA in Europe was clear and manifest in the renounce of war compensations and the establishment of the European Recovery Program, the Marshall plan for the reconstruction of Europe. Production in Western Europe was so successfully recovered in the beginning of 1950s that those goods that were once made in USA started to be made in Europe by American companies and imported in USA (Kenen 1967). In this historical context of dependency of Germany and Western Europe on USA, shocks occurred in the US economy were likely propagated to Europe rather than the other way around. In other words, statistical analysis of data should show Germany and USA to be respectively a 'small' and a 'big' country during the Bretton-Woods period.

This analysis takes also up other issues concerning the categories of price indices and interest rates that could be used. Often it is opted for the CPI but other price indices can also be chosen such as for instance unit labor costs etc.; in this chapter only PPI are chosen.

The database consists of the producer price indices p_t and p_t^*, long bond yield (10 years) i_t^l and i_t^{l*}, the Frankfurt interbank offered rate for the Bretton-Woods period and three month German treasury bill rates[5] i_t^s, the three month US Treasury bill rate $i_t^{s^*}$ and the spot exchange rate USdollar/Deutschemark s_t. The Frankfurt interbank offered rate i_t^s was chosen as we could not have the three month Treasury bill rate for Germany. The particular type of data used for the short term interest rate for Germany show a recurrent anomalous cyclicality between September and November that we cannot explain. Using seasonal and other dummies in the model we hope to have partially cleaned a bit the data but great caution should be paid in interpreting the results of the analysis when we include the short term interest rates. As short term interest rate can be steered by the central bank, the German short term interest rate we used should reflect the respective discount rate, as it seems to do except for the above-mentioned months.

This database was extracted from Datastream and its sources are the Financial Statistics(IFS) publication of the IMF, the publication 'Main Economic Indicators' of the OECD and national government institutes but data on prices with four decimals were provided by Prof. Juselius. Data are monthly, not seasonally adjusted.[6]. Prices and exchange rate are taken in natural logs, the yearly interest rates in percentage and divided by 12 to obtain monthly rates. We transformed prices and the exchange rate with their natural log, the yearly interest rates

[5] It is an interbank interest rate. More precisely, it refers to an interest rate, determined at the Frankfurt Banking Centre, at which banks may invest Deutschmark deposits with other banks for a period of 3 to 6 months, in the form of fixed or time deposits.

[6] In this paper the effects from seasonality are removed using centered seasonal dummies that sum to zero over each year (see Johansen 1995 p. 84 for further details). Motivations for using not seasonally adjusted time series in cointegration analysis are found in Johansen 1995.

were taken in percentage (i.e. divided by 100) and divided by 12 to obtain the monthly rates while *ppp* was divided by 100.

3.2 A multivariate time series analysis for the Post Bretton-Woods period

We needed the following dummy variables:

$$\begin{bmatrix} DS7986, \Delta DS7911, \Delta DS8610, D7912, Di8003, D8005, D8007, \\ D8011, D8101, D8103, Di8105, D8110, Di8111, D8203, D8208, \\ D8411, Di8412, D8604, D8808, D8902, D9008, D9102, D9601 \end{bmatrix}$$

where:

$Dixx.yy$ is 1 at $19xx.yy_t$, -1 at $19xx.yy_{t+1}$ and 0 otherwise measuring a transitory shock.

$Dxx.yy$ is 1 at $19xx.yy_t$ and 0 otherwise measuring a permanent intervention shock.

$DS7986$ is 1 from November 1979 till October 1986 and zero otherwise. $DS7986$ aims to capture the structurally different regime of the period characterized a restrictive monetary policy. This agrees, we think, with the findings by Hansen and Johansen in 1999 that at least part of the period, 1979-1982, defined a structural different regime (Juselius and MacDonald 2003).

$\Delta DS79.11$ and $\Delta DS86.10$ are $\Delta DS7986$ measured respectively in November 1979 and October 1986 and serve to remove the permanent effect generated by the shift dummy.

We decided for a model with three lags and a cointegration rank equal to three as both the trace test and the analysis of the eigenvalues of the companion matrix supported the rank restriction of the Π matrix $r = 3$.

Single cointegration hypothesis

$\mathcal{H}_1$ to $\mathcal{H}_9$ are hypothesis on pairs of variables, such as relative inflation ($\mathcal{H}_1$), relative interest rates ($\mathcal{H}_2, \mathcal{H}_3$), stationary real interest rates ($\mathcal{H}_4, \mathcal{H}_5, \mathcal{H}_6$ and $\mathcal{H}_7$) (Tab. 1) and the spread between interest rates ($\mathcal{H}_8, \mathcal{H}_9$). Although some were accepted, the $p-values$ were not very high.

$\mathcal{H}_{10}$ is a combination of $\mathcal{H}_1$ with $\mathcal{H}_2$, $\mathcal{H}_{11}$ is a combination of $\mathcal{H}_4$ with $\mathcal{H}_5$, $\mathcal{H}_{13}$ is a combination of $\mathcal{H}_1$ with $\mathcal{H}_3$, $\mathcal{H}_{14}$ is a combination of $\mathcal{H}_6$ with $\mathcal{H}_7$. In these cases, but $\mathcal{H}_{13}$, the

$p - values$ are not very high. $\mathcal{H}_{12}$ and $\mathcal{H}_{15}$ may be considered as the *uip* condition. Support for *uip* is not very evident although using the short term interest rates the hypothesis would be accepted with a $p - value$ of 0.20. $\mathcal{H}_{16}$ and $\mathcal{H}_{17}$ combine $\mathcal{H}_2$ with $\mathcal{H}_3$, i.e. the spread among interest rates between the two countries. $\mathcal{H}_{17}$ can also be seen as a combination of the term spreads ($\mathcal{H}_8$ and $\mathcal{H}_9$). Both $\mathcal{H}_{16}$ and $\mathcal{H}_{17}$ are rejected.

$\mathcal{H}_{18}$ to $\mathcal{H}_{26}$ combine the pairs of variables described from $\mathcal{H}_1$ to $\mathcal{H}_9$ with *ppp*. With the exception of the long term interest rate spread, combining these parities with the *ppp* does not produce more significant stationary relationships.

$\mathcal{H}_{27}$ instead combines the *uip* condition shown in $\mathcal{H}_{12}$ with the *ppp* producing a stationary relation accepted with a $p - value$ of 0.30. $\mathcal{H}_{29}$, accepted with a $p - value$ of 0.72, describes a homogeneous relationship (that is coefficients sum to zero) between German and US inflation and the German bond rate, capturing the effects of imported inflation from the US to Germany. As it was for the small model, it is interesting to note that notwithstanding producer price indices do not include prices for imported goods, both the producer price and the consumer price indices have very similar estimated parameters (they are exactly 1, -0.34 and -0.66 in Juselius and MacDonald 2003!).

$\mathcal{H}_{27}$ can be interpreted as:

- A linear long-run relationship between *ppp* and *uip*:

$$(-\Delta p_t + \Delta p_t^*) + (i_t^l - i_t^{l*}) = \omega ppp_t, \text{ i.e. } -uip_t = \omega ppp_t.$$

- The log of real exchange rate proportional to the spread between the real interest rates in the two countries:

$$(i_t^l - \Delta p_t) - (i_t^{l*} - \Delta p_t^*) = \omega ppp_t.$$

- An equation for the determinants of the exchange rate that shows the nominal exchange rate in function of the spread of prices and the spread of real interest rates:

$$s_t = (p_t - p_t^*) + \tfrac{1}{\omega}(i_t^l - \Delta p_t) - \tfrac{1}{\omega}(i_t^{l*} - \Delta p_t^*).$$

- An international real interest rate parity which shows that the US real interest rate is lower than the German real interest rate when *ppp* is positive and the US real interest rate increases when ppp is negative, i.e. when the US prices are greater than German prices:

$$(i_t^{l*} - \Delta p_t^*) = (i_t^l - \Delta p_t) - \omega ppp_t.$$

A very similar relation was found by Juselius and MacDonald (2000) using consumer price indices. This shows a remarkable robustness of the validity of the relation found by Juselius and MacDonald to changes in price indices.

$\mathcal{H}_{28}$ is the restricted third cointegration relation we were trying to find. It can be interpreted in many ways as it combines $\mathcal{H}_1$,$\mathcal{H}_2$ and $\mathcal{H}_3$, $\mathcal{H}_{17}$ and $\mathcal{H}_1$, $\mathcal{H}_{15}$ and $\mathcal{H}_2$, $\mathcal{H}_{24}$,$\mathcal{H}_{23}$ and $\mathcal{H}_2$ or other hypothesis. Thus, $\mathcal{H}_{28}$ can be seen as:

$$\left(i_t^l - i_t^{l*}\right) - (i_t^s - i_t^{s*}) = -\left(\Delta p_t - \Delta p_t^*\right) \tag{24}$$

which shows that if the spread between actual domestic and foreign inflation is non stationary, then the spread between domestic and foreign yield gap would also have to be non stationary. Alternatively $\mathcal{H}_{28}$ may be interpreted as:

$$(i_t^{s*} - \Delta p_t^*) = (i_t^s - \Delta p_t) - \left(i_t^l - i_t^{l*}\right) \tag{25}$$

which shows the short term real interest rate parity as a stationary relation whenever the long term bond spread were stationary. $\mathcal{H}_{28}$ is accepted with a $p - value$ of 0.85.

Tab. 1: Cointegration relations

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	i_t^s	i_t^{s*}	ppp_t	$DS7986$	$constant$	$\chi^2(\nu)$	$p-val$
$\mathcal{H}_1$	1	-1	0	0	0	0	0	-0.004	0.000	7.77 (4)	0.10
$\mathcal{H}_2$	0	0	1	-1	0	0	0	0.002	-0.000	19.50 (4)	0.00
$\mathcal{H}_3$	0	0	0	0	1	-1	0	0.002	-0.000	15.69 (4)	0.00
$\mathcal{H}_4$	1	0	-1	0	0	0	0	0.000	0.000	6.37 (4)	**0.17**
$\mathcal{H}_5$	0	1	0	-1	0	0	0	0.005	-0.000	11.45 (4)	0.02
$\mathcal{H}_6$	1	0	0	0	1	0	0	-0.003	0.000	21.61 (4)	0.00
$\mathcal{H}_7$	0	1	0	0	0	1	0	0.006	0.000	10.14 (4)	0.04
$\mathcal{H}_8$	0	0	1	0	-1	0	0	-0.001	-0.000	25.95 (4)	0.00
$\mathcal{H}_9$	0	0	0	1	0	-1	0	0.000	-0.000	6.89 (4)	**0.14**
$\mathcal{H}_{10}$	1	-1	-0.421	0.421	0	0	0	-0.004	0.000	6.98 (3)	0.07
$\mathcal{H}_{11}$	1	-0.276	-1	0.276	0	0	0	-0.001	0.000	5.34 (3)	**0.15**
$\mathcal{H}_{12}$	1	-1	-1	1	0	0	0	-0.006	-0.000	8.20 (4)	0.08
$\mathcal{H}_{13}$	1	-1	0	0	-0.576	0.576	0	-0.005	0.000	3.08 (3)	**0.38**
$\mathcal{H}_{14}$	1	-1.401	0	0	-1	1.401	0	-0.008	-0.000	5.33 (3)	**0.15**
$\mathcal{H}_{15}$	1	-1	0	0	-1	-1	0	-0.006	0.000	6.03 (4)	**0.20**
$\mathcal{H}_{16}$	0	0	1	-1	-0.817	0.817	0	0.000	-0.000	11.88 (3)	0.01
$\mathcal{H}_{17}$	0	0	1	-1	-1	-1	0	-0.000	-0.000	12.26 (4)	0.02
$\mathcal{H}_{18}$	1	-1	0	0	0	0	-0.673	-0.006	-0.000	6.84 (3)	0.08
$\mathcal{H}_{19}$	0	0	1	-1	0	0	-0.669	0	-0.000	2.42 (4)	**0.16**
$\mathcal{H}_{20}$	0	0	0	0	1	-1	-2.015	-0.005	–0.000	7.51 (3)	0.06
$\mathcal{H}_{21}$	1	0	-1	0	0	0	0.056	0.000	0.000	6.33 (3)	0.10
$\mathcal{H}_{22}$	0	1	0	-1	0	0	-1.182	0.001	-0.000	8.17 (3)	0.04
$\mathcal{H}_{23}$	1	0	0	0	-1	0	7.693	0.030	-0.000	12.92 (3)	0.00
$\mathcal{H}_{24}$	0	1	0	0	0	-1	-0.556	0.004	0.000	9.54 (3)	0.02
$\mathcal{H}_{25}$	0	0	1	0	-1	0	7.438	0.029	0.000	13.01 (3)	0.00
$\mathcal{H}_{26}$	0	0	0	1	0	-1	0.337	0.001	-0.000	5.78 (3)	**0.12**
$\mathcal{H}_{27}$	1	-1	-1	1	0	0	1.430	0	0.000	4.88 (4)	**0.30**
$\mathcal{H}_{28}$	1	-1	1	-1	-1	1	0	-0.004	-0.000	1.34 (4)	**0.85**
$\mathcal{H}_{29}$	1	-0.360	-0.640	0	0	0	0	-0.001	0.000	1.34 (3)	**0.72**

The ppp term has been divided by 100

Fully specified cointegrating relations

In Tab. 2 a structural representation of the cointegration space is finally given. The fully specified cointegrating relations were tested with the LR test procedure in Johansen and Juselius (1994) and accepted with a $p-value$ of 0.79.

The adjustment coefficients are also reported. None of the adjustment parameters are significant for the long term interest rates, suggesting they are the weakly exogenous variables that push the system while some of the adjustment parameters referring to *ppp* are significant meaning that the weak exogeneity for *ppp* is less evident in the extended than in the small model. Restricting to zero the adjustment parameters for the German and US long term interest rate the hypothesis were respectively accepted with a $p-value$ of 0.90 and 0.76. Restricting both, the $p-value$ was 0.85 (incidentally the same value of Juselius and MacDonald 2003 for similar restrictions). Restricting to zero the adjustment parameters for the long term interest rates and *ppp* the hypothesis was accepted with a $p-value$ of 0.48, while restricting for the adjustment parameters just for *ppp* was accepted with a $p-value$ of 0.39. Other restrictions to α produced either very low $p-values$ for the German short term interest rate (0.12) or $p-values$ were close to zero.

Tab. 2: A structural representation of the cointegration space (extended model)

	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$		$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$
Δp_t	1	1	1	$\Delta^2 p_t$	**−0.854** (−7.3)	0.058 (0.7)	**0.272** (3.2)
Δp_t^*	**−0.368** (6.92)	−1	−1	$\Delta^2 p_t^*$	**−0.435** (−2.0)	**0.446** (2.9)	**0.323** (2.0)
i_t^l	**−0.632** (6.56)	−1	1	Δi_t^l	0.006 (0.7)	0.001 (0.2)	−0.005 (−0.7)
i_t^{l*}	0	1	−1	Δi_t^{l*}	−0.004 (−0.3)	0.000 (0.0)	−0.006 (−0.6)
i_t^s	0	0	−1	Δi_t^s	0.003 (0.3)	**−0.018** (−2.9)	**0.021** (3.2)
i_t^{s*}	0	0	1	Δi_t^{s*}	**0.034** (2.7)	**0.021** (2.4)	**−0.042** (−4.5)
ppp_t^1	0	**1.420** (6.56)	0	Δppp_t	0.002 (0.1)	**−0.034** (−2.8)	**0.025** (2.0)
$DS7986$	**−0.001** (4.85)	0	**−0.003** (−2.46)				
$constant$	0.000	−0.000	−0.000				

The ppp term has been divided by 100

Common trends

We report the VMA (common trends) representation for two different cases based on the fully specified cointegrating relations restricted VAR model for $r = 3$ after having fully specified cointegration relations with weak exogeneity of i_t^l, i_t^{l*} imposed on α. The other two driving forces beyond long term interest rates, may be further searched among *ppp* and short term interest rates or a combination of these.

The estimates of the C matrix in Tab. 3 measure the total impact of permanent shocks to each of the variables on all other variables. A row of the C matrix gives an indication of which variables have been particularly important for the stochastic trend behavior of the variable in the row.

Tab. 3: The estimates of the long run impact matrix C

C	$\sum \hat{\varepsilon}_{i_t^l}$	$\sum \hat{\varepsilon}_{i_t^{l*}}$	$\sum \hat{\varepsilon}_{i_t^s}$	$\sum \hat{\varepsilon}_{i_t^{s*}}$	$\sum \hat{\varepsilon}_{ppp_t}$
Δp_t	**0.99** 3.87	0.31 1.63	−0.20 −1.04	**0.50** 2.94	**0.64** 3.45
Δp_t^*	0.52 1.08	0.37 1.01	**-0.75** −2.00	**1.11** 3.47	**1.68** 4.75
i_t^l	**1.28** 6.00	0.28 1.73	0.14 0.85	0.10 0.72	−0.02 −0.11
i_t^{l*}	−0.18 −0.65	**1.19** 5.73	0.35 1.62	0.27 1.50	−0.02 −0.10
i_t^s	**1.05** 3.59	−0.11 −0.48	**1.00** 4.40	0.33 1.68	**-0.50** −2.34
i_t^{s*}	**-0.88** −2.53	**0.86** 3.31	**0.66** 2.46	**1.12** 4.86	0.53 2.09
ppp_t	**0.70** 3.10	**-0.61** −3.55	**-0.53** −3.00	**0.31** 2.09	**0.73** 4.40

The C matrix suggests that:

- Inflation rates are adjusting.[7]

- German inflation rate is pushed by home interest rates and indirectly by long term US interest rates through US short term interest rate and by *ppp*.

- US inflation is not pushed by the German interest rates but by US short term interest rates which is pushed by US long term interest rate and by *ppp*.

[7]The columns corresponding to $\sum \hat{\varepsilon}_{\Delta p}$ and $\sum \hat{\varepsilon}_{\Delta p^*}$ are not shown as no value was found significant.

- Shocks to long term interest rates have significant effects on short term interest rates, but not the other way round.

- Shocks to short term interest rates had a significant effect on inflation.

- Shocks to the US long term interest rate have an impact on both the German and US inflation rates.

- Shocks to *ppp* affect the inflation rates in the two countries.

With regard to the role of the short term interest rates, it seems that short term interest rates were significantly important for inflation rates at least in Germany and in the USA, signaling, in principle, the possibility to influence inflation rates steering the short term interest rates. The central bank, being the monopoly supplier of the monetary base is able to influence money market condition and steer short term interest rates. A change in money market interest rates would set in motion a number of mechanisms and actions by economic agents influencing inflation through the monetary policy transmission mechanism (ECB 2004). Our results agree with the view of the ECB, however, the results for short term interest rates show also a significant reaction to long term interest rates. From Tab. 3 appears rather clearly that effects go from bond rates influencing treasury bill rates, influencing inflation rates as Juselius and MacDonald (2003) put forward. Thus, although, monetary policy may steer inflation rates via short term interest rates, this analysis shows that long term interest rates, and with it the perspectives of both growth and inflation, affect significantly short term interest rates, hence, inflation rates.

3.3 A multivariate time series analysis for the Bretton-Woods period

We needed the following dummy variables for the extended model:

$$\left[\ DS61.03, \Delta DS61.03, D60.06, Di61.09, D66.12, D67.07\ \right]$$

$DS61.03$ is 1 since March 1961 and zero otherwise. $DS61.03$ takes into account an important official change in exchange rate of the German Mark vs. the US Dollar.

We decided for two lags and a cointegration rank equal to four as the analysis of eigenvalues of the companion matrix supported the hypothesis the rank restriction $r = 4$.

Single cointegration hypothesis

$\mathcal{H}_1$ to $\mathcal{H}_7$ (Tab. 4) are hypothesis on single variables. Inflation rates for both countries turned to be stationary with high $p - values$ ($\mathcal{H}_1$and $\mathcal{H}_2$). Relative *ppp* is logically accepted as it turns out to be a linear combination of two stationary inflation rates. Stationarity in inflation rates implies that: *i*) prices are most likely $I(1)$; *ii*) the *ppp* could only be satisfied only in the case of cointegration between prices. This shows that the Bretton-Woods system planned to be mild inflationary proved to guarantee stability in inflation rates. Both the short and long term interest rates and *ppp* turned out to be non stationary.

$\mathcal{H}_8$ to $\mathcal{H}_{11}$ are hypothesis on a pair of variables. $\mathcal{H}_8$ and $\mathcal{H}_9$ are hypothesis on the relative interest rates. Cointegration between US and German interest rates is excluded. $\mathcal{H}_8$ and $\mathcal{H}_9$ can be interpreted also as a hypothesis on the *uip* parity: since during the Bretton-Woods regime the exchange rates were fixed against the US dollar, the expected change of the exchange rates could not be anything but equal to zero $E_t\Delta_m s_{t+m} = 0$. The *uip* reduces to $-i_t + i_t^* = 0$ and if the *uip* holds empirically, $-i_t + i_t^* \sim I(0)$, otherwise $-i_t + i_t^* \sim I(1)$. Evidence shown in Tab. 4 points out the *uip* does not hold during the Bretton-Woods period. $\mathcal{H}_{10}$ and $\mathcal{H}_{11}$ are rejected hypothesis on the spread between interest rates.

$\mathcal{H}_{12}$ and $\mathcal{H}_{13}$ combine $\mathcal{H}_8$ with $\mathcal{H}_9$, i.e. the spread among interest rates between the two countries. $\mathcal{H}_{13}$ can also be seen as a combination of the term spreads ($\mathcal{H}_{10}$ and $\mathcal{H}_{11}$). Both $\mathcal{H}_{12}$ and $\mathcal{H}_{13}$ are accepted with rather high $p - value$. $\mathcal{H}_{13}$ is the restricted third cointegration relation we were trying to find. It can be interpreted in many ways:

- As inflation rates are found stationary ($\mathcal{H}_1$and $\mathcal{H}_2$), $\mathcal{H}_{13}$ can be seen as:

$$\left(i_t^l - i_t^{l*}\right) - \left(i_t^s - i_t^{s*}\right) = -\left(\Delta p_t - \Delta p_t^*\right) \tag{26}$$

which shows that when the spread between actual domestic and foreign inflation is stationary, then the spread between domestic and foreign yield gap would also has to be stationary.

- Alternatively $\mathcal{H}_{13}$ may be interpreted as:

$$\left(i_t^{s*} - \Delta p_t^*\right) = \left(i_t^s - \Delta p_t\right) - \left(i_t^l - i_t^{l*}\right)$$

which shows the short term real interest rate parity as a stationary relation if the long term bond spread would be stationary $\mathcal{H}_{13}$ is accepted with a $p-value$ of 0.66.

This very same relation was found to hold for the Post Bretton-Woods period. What changes is the degree of integration of the inflation rates, not the relationships. Simplifying for the inflation rates the last two relationships reduce to:

$$\left(i_t^l - i_t^{l*}\right) - \left(i_t^s - i_t^{s*}\right) \sim I\left(0\right)$$

which shows that when the spread between domestic and foreign yield gap would also has to be stationary and

$$i_t^{s*} = i_t^s + \left(i_t^l - i_t^{l*}\right) \sim I\left(0\right)$$

which shows the nominal short term interest rate parity as a stationary relation whenever the nominal long term bond spread were stationary. These last equations could not be supported during the Post Bretton-Woods period because inflation rates for that period were neither stationary or cointegrating.

$\mathcal{H}_{14}$ combines the *uip* condition shown in $\mathcal{H}_8$ with the *ppp* producing a stationary relation accepted with a $p-value$ of 0.16.

Tab. 4: Cointegration relations

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	i_t^s	i_t^{s*}	ppp_t	$DS61.03$	$constant$	$\chi^2(\nu)$	$p-val$
$\mathcal{H}_1$	1	0	0	0	0	0	0	0.002	-0.002	2.01 (3)	**0.57**
$\mathcal{H}_2$	0	1	0	0	0	0	0	0.001	-0.001	2.81 (3)	**0.42**
$\mathcal{H}_3$	0	0	1	0	0	0	0	0.007	-0.009	10.06 (3)	0.02
$\mathcal{H}_4$	0	0	0	1	0	0	0	0.001	-0.004	7.20 (3)	0.07
$\mathcal{H}_5$	0	0	0	0	1	0	0	0.003	-0.006	9.40 (3)	0.02
$\mathcal{H}_6$	0	0	0	0	0	1	0	0.001	-0.003	6.95 (3)	0.07
$\mathcal{H}_7$	0	0	0	0	0	0	1	-0.001	0.010	9.08 (3)	0.03
$\mathcal{H}_8$	0	0	1	-1	0	0	0	-0.003	-0.000	9.91 (3)	0.02
$\mathcal{H}_9$	0	0	0	0	1	-1	0	-0.001	-0.000	9.30 (3)	0.03
$\mathcal{H}_{10}$	0	0	1	0	-1	0	0	-0.001	-0.001	6.50 (3)	0.09
$\mathcal{H}_{11}$	0	0	0	1	0	-1	0	-0.001	-0.000	7.15 (3)	0.07
$\mathcal{H}_{12}$	0	0	1	-1	-1.013	1.013	0	-0.001	-0.000	1.59 (2)	**0.45**
$\mathcal{H}_{13}$	0	0	1	-1	-1	1	0	-0.001	-0.000	1.60 (3)	**0.66**
$\mathcal{H}_{14}$	0	0	1	-1	0	0	-4.078	0.003	-0.040	3.61 (2)	**0.16**

The ppp term has been divided by 100

Fully specified cointegrating relations

In Tab. 5 a structural representation of the cointegration space is finally given. The fully specified cointegrating relations were tested with the LR test procedure in Johansen and Juselius (1994) and accepted with a $p-value$ of 0.42.

The adjustment coefficients are also reported. There is only one adjustment parameters boundary significant for the US long term interest rates, suggesting it might be a weakly exogenous variables that pushes the system while some of the adjustment parameters referring to *ppp* are significant meaning that the weak exogeneity for *ppp* is less evident in the extended than in the small model. Restricting to zero the adjustment parameters for US long term interest rate the hypothesis is still accepted with a $p-value$ of 0.38.

TAB. 5: A STRUCTURAL REPRESENTATION OF THE COINTEGRATION SPACE (EXTENDED MODEL)

	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$		$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$
Δp_t	1	0	0	0	$\Delta^2 p_t$	**−0.769** −4.6	−0.068 −0.4	−0.630 −1.5	−0.590 −1.1
Δp_t^*	0	1	0	0	$\Delta^2 p_t^*$	−0.123 0.5	**−0.864** −4.0	−0.189 −0.3	**2.560** 3.4
i_t^l	0	0	1	1	Δi_t^l	**0.019** 3.2	−0.002 −0.4	−0.020 −1.3	0.003 0.1
i_t^{l*}	0	0	−1	−1	Δi_t^{l*}	0.003 0.4	−0.006 −0.9	**0.044** 2.5	0.041 1.8
i_t^s	0	0	0	−1	Δi_t^s	0.015 1.0	0.026 1.8	0.007 0.2	**0.154** 3.0
i_t^{s*}	0	0	0	1	Δi_t^{s*}	**−0.019** 2.4	**0.025** 3.3	0.005 0.3	−0.012 −0.5
ppp_t^1	0	0	**−3.263** −7.33	0	Δppp_t	−0.001 −0.4	−0.003 −1.1	0.004 0.6	**−0.031** −3.2
$DS61.03$	**0.002** 9.55	**0.001** 4.65	**0.002** 6.30	**−0.001** −5.29					
constant	−0.002	−0.000	−0.033	−0.000					

The ppp term has been divided by 100

Common trends

We report the VMA (common trends) representation for two different cases: (i) based on the rank restricted VAR model for $r = 4$ and having fully specified cointegrating relations (ii) based on (i) but imposing weak exogeneity of i_t^{l*} imposed on α.

The estimates of the C matrix in Tab. 6 measure the total impact of permanent shocks to each of the variables on all other variables. A row of the C matrix gives an indication of which variables have been particularly important for the stochastic trend behavior of the variable in the row.

TAB. 6: THE ESTIMATES OF THE LONG RUN IMPACT MATRIX C

C	$\sum \hat{\varepsilon}_{\Delta p_t}$	$\sum \hat{\varepsilon}_{\Delta p_t^*}$	$\sum \hat{\varepsilon}_{i_t^l}$	$\sum \hat{\varepsilon}_{i_t^{l*}}$	$\sum \hat{\varepsilon}_{i_t^s}$	$\sum \hat{\varepsilon}_{i_t^{s*}}$	$\sum \hat{\varepsilon}_{ppp_t}$	$\sum \hat{\varepsilon}_{i_t^{l*}}$
Δp_t	0	0	0	0	0	0	0	0
Δp_t^*	0	0	0	0	0	0	0	0
i_t^l	0.030 1.95	−0.012 −1.25	**0.894** 2.19	**0.767** 3.11	0.096 0.47	−0.160 −0.63	0.058 0.05	**1.129** 3.30
i_t^{l*}	0.009 0.45	−0.002 −0.17	1.090 1.93	**0.817** 2.39	−0.514 −1.80	0.557 1.58	−1.951 −1.13	**1.470** 4.59
i_t^s	0.006 0.22	0.024 1.44	1.245 1.79	0.330 0.79	0.136 0.39	**1.193** 2.77	2.552 1.21	1.051 1.72
i_t^{s*}	−0.015 −0.48	0.033 1.73	1.441 1.76	0.379 0.77	−0.473 −1.14	**1.910** 3.74	0.543 0.22	**1.392** 2.36
ppp_t	0.006 0.92	−0.003 −0.70	−0.060 −0.33	−0.015 −0.14	**0.187** 2.03	−0.220 −1.94	0.616 1.11	−0.122 −1.114

From the C matrix we note that:

- Cumulative shocks to inflation rates, which were found to be and modeled as stationary variables in the restricted model, have obviously no significant long run impact on any other variable in the unrestricted VAR model.
- Cumulative shocks to the US short and long term interest rates are found significant.
- Cumulative shocks to the US long term interest rate have a significant impact on the German long term interest rate.
- Cumulative shocks to the US short term interest rate have a significant impact on the German short term interest rate.
- Cumulative shocks to the German interest rated do not have significant effects on the other variables of the system.

  This result emphasizes the evidence that the German long term interest rate was pushed from the USA. Imposing weak exogeneity for the US long term interest rate we find that: cumulative shocks to the US long term interest rate have a significant impact on the German long term and the US short term interest rates. However the evidence that the US short term interest rate was driven by the long term interest rate is less evident than it was found in other studies referring to the Post Bretton-Woods period.

4 Conclusions

The main result of this chapter is that important cointegration relationships found to hold for the Post Bretton-Woods essentially hold for the Bretton-Woods period as well. We think that this is a remarkable result because the two periods were characterized by distinct exchange rate regimes and a different regulation in capital markets.

It appears that the relationships found to hold for the Bretton-Woods period are a particular case of the relationships that hold for the Post Bretton-Woods period. In both periods a linear long-run relationship between *ppp* and *uip*, namely $uip_t + \omega ppp_t \sim I(0)$ holds, so that $\Delta p_t - \Delta p_t^* - i_t^l + i_t^{l*} + \omega ppp_t \sim I(0)$. However the pegged exchange rate system seemed to ensure stationary inflation rates so that the simplified stationary relation $-i_t^l + i_t^{l*} + \omega ppp_t \sim I(0)$ also holds for the Bretton-Woods period.

Similarly the relationships $\left(i_t^l - i_t^{l*}\right) - \left(i_t^s - i_t^{s*}\right) = \left(\Delta p_t - \Delta p_t^*\right)$, which shows that when the spread between actual domestic and foreign inflation is stationary then the spread between domestic and foreign yield gap would also has to be stationary, and $\left(i_t^{s*} - \Delta p_t^*\right) = \left(i_t^s - \Delta p_t\right) + \left(i_t^l - i_t^{l*}\right)$, which shows the short term real interest rate parity as a stationary relation if the long term bond spread would be stationary, hold for both periods. However because of the stationary inflation rates in the Bretton-Woods period, simplifying, the two relationships reduce to $\left(i_t^l - i_t^{l*}\right) - \left(i_t^s - i_t^{s*}\right) \sim I(0)$, which shows that when the spread between domestic and foreign yield gap would also has to be stationary, and $i_t^{s*} = i_t^s + \left(i_t^l - i_t^{l*}\right) \sim I(0)$, which shows the nominal short term interest rate parity as a stationary relation whenever the nominal long term bond spread were stationary.

Different values of the parameter ω between the two periods were estimated. We maintain that the parameter ω might be interpreted as the responsiveness of the capital movements that enter in the capital and financial account to *uip*. A small value of the parameter ω may imply a large responsiveness of capital movements to the net interest rate differential. Due to the restrictions and the heavy regulation for capital movements typical of the Bretton-Woods period, we expected a slower responsiveness of capital movements to the net interest rate differential, then a higher value of the parameter ω for the Bretton-Woods period than the recent floating exchange rate experience. In fact, it was found that ω was between 2 and 6 times greater during the Bretton-Woods than the Post Bretton-Woods period.

We maintained that statistical analysis of data should have shown Germany and USA to be respectively a 'small' and a 'big' country. In this respect we found that cumulative shocks to the US long term interest rate have a significant impact on the German long term interest rate, cumulative shocks to the US short term interest rate have a significant impact on the German short term interest rate and cumulative shocks to the German interest rated do not have significant effects on the other variables of the system. This result emphasizes the evidence that the German long term interest rate was indeed pushed from the USA. Imposing weak exogeneity for the US long term interest rate we find that: cumulative shocks to the US long term interest rate have a significant impact on the German long term and the US short term interest rates. However the evidence that the US short term interest rate was driven by the long term interest rate is less evident than it was found in other studies referring to the Post Bretton-Woods period.

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