Do trend traders tame the chaos? - Feedback provides stability.

Preliminary version - please do not quote

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Abstract

This paper concentrates on stability properties of heterogeneous agent models which include trend traders. So far, papers have only described the models' behaviour in the very long run. I propose an explanation for the phenomenon that computer simulations of these models regularly converge if certain parameter constellations are used. In particular, insights of physics concerning time-delayed feedback control are fruitful for a deeper understanding of this event. I apply this theoretical knowledge about chaos control to the current model by De Grauwe and Grimaldi (2006) as an example. This paper shows that the way trend traders form their expectations serves as a feedback rule stabilising the system. Extrapolating trends from the past then tames the chaos in the model.

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1 Introduction

In recent years, agent-based computational economics (ACE) enjoys growing popularity among researchers.[1] As this approach allows for heterogeneous agents, who apply different decision rules, researchers can study the interaction between single agents and its effect on the market outcome. Even more, with computer simulations it is possible to examine the model's stability properties in the very long run. Clearly, this represents a great advantage over empirical studies, where we regularly face constraints concerning the data available. However, despite its methodological appeal, ACE also contains some drawbacks. These concern the relationship between the model set-up and its convergence behaviour. So far, papers have only *stated* that models converged. The reasons for many models to stabilise over time have not been in the focus of economic research. This is quite surprising as gaining knowledge about *why* many models converge after a certain time, would undoubtedly lead to a better understanding of the model itself and its underlying processes.

With this paper, I therefore aim at closing the gap. In the following, results from chaos theory in physics are used in order to explain the fact that most models stabilise under certain conditions. To ease understanding, I apply the theoretical results to a typical heterogeneous agent model. It is a recent model for the foreign currency market by De Grauwe and Grimaldi (2006), which exemplifies the issue. Their model includes, amongst other types of agents, trend-traders, who speculate on the continuity of past developments. Such trend traders are widely used in the literature and are part of nearly every heterogeneous agent model. Trend traders, by definition, extrapolate past variations into the future and make their decisions upon this grounding. I will show that under certain conditions the behaviour of these agents stabilises the system in an indirect way - extrapolating trends from

[1] For instance, Brock/Hommes (1997, 1998), Brock/LeBaron (1996), Chiarella (1992), De Grauwe/Grimaldi (2006), De Long/Shleifer/Summers/Waldmann (1990), Frankel/Froot (1988), Hommes (2005a), Lux (1997). A good overview is provided in Hommes (2005b) and LeBaron (2005).

the past tames the chaos in the model.

The remainder of the paper is organised in the following way. In section 2 I shortly present methods which are used to control chaotic behaviour in physics. The main part of the paper will be to lay out the theoretical framework of time-delayed feedback control and to apply this concept to an economic example. Section 4 concludes.

2 Chaos control in physics

Before turning to the analysis, let me give a brief introduction into chaos control, which is a prominent exploratory focus in physics. There has been done, unlike in economics, considerable work on the comprehension and the control of chaotic systems. By a "chaotic" system, physicists mean a system, whose outcome crucially depends on its initial conditions (sensitivity to initial conditions).[2] As a result of this sensitivity, the behavior of models that exhibit chaotic movements appears to be random, even though the model itself is deterministic in the sense that it does not contain any random parameters. This is also true for the deterministic version of the model by De Grauwe and Grimaldi (2006) as well as for heterogeneous agent models in general.

In the last years, physicists did not only describe the chaotic behaviour of these systems but redirected their interest to ways of their stabilisation. This triggered the development of the research area of chaos control. The origin of this approach lies in the observation that on the one hand chaotic motion provides a huge number of unstable states and that on the other hand each of these states can be stabilised by extremely small control forces. As substantial progress was achieved during the last decade, these insights of physics can serve as a starting point for an economic debate about how stability properties of heterogeneous agent models arise.

[2] A well-known example of this sensitivity is the butterfly effect where the flapping of a butterfly's wings produces tiny changes in the atmosphere which over time lead to dramatic effects such as tornados.

Physicists developed three general concepts in order to control chaotic systems. First, by means of a continuous external perturbation, which constantly forces the varying variable back on a smooth path. Second, by a time-discrete conditioned intervention, where deviations are corrected once in a while. Third, and more interesting from my point of view is the so-called time-delayed feedback control. This method, which is due to Pyragas (1992), is presented in more detail in the following section.

3 Do trend traders tame the chaos?

3.1 Time-delayed feedback control in physics

Amongst physicists conducting research on chaos control, it had been common knowledge for a long time that time delay does not increase but reduce the efficiency of a control scheme. This is straight forward as intermittent corrections are naturally less precise and thus less powerful than continuous ones. However, contrary to what one would expect intuitively, time delay can also be used to stabilise chaotic movements. Such a time-delayed feedback control was first suggested for physical problems by Pyragas in 1992. His approach uses a measurable output signal s_t for stabilisation purposes. On the basis of this signal at different moments in time, physicists create a feedback variable $\Delta s_{t-\tau}^{feed}$. This feedback is the difference between the current state of the system and its state some τ time units ago. Loosely speaking, one can imagine the step of deducing the feedback variable as collecting information about how strongly and in which intervals the system fluctuates.

Formally this proceeding look as follows:

$$\Delta s_{t-\tau}^{feed} = s_t - s_{t-\tau}. \tag{1}$$

Once it is clear how the system evolves, this feedback $\Delta s_{t-\tau}^{feed}$ is linearly amplified by a model specific parameter value ϱ, which weights the fluctuation pattern in order to generate the actual control force.

$$F_t = \varrho[s_t - s_{t-\tau}] = \varrho \Delta s_{t-\tau}^{feed}. \tag{2}$$

Pyragas showed in his paper that by re-introducing F_t into the chaotic system, its chaotic behaviour is swept away. His finding is based on experimental evidence. These experimental results are supported by a numerical analysis of the Lyapunov exponent λ, which is a well-established quantitative measure of the sensitivity to initial conditions in chaos theory. This concept of time-delayed feedback control can thus suggest how an experimental set-up or a model may be modified to obtain a converging outcome. It is worthwhile pointing out that above all the time-difference τ and the multiplier ϱ are sufficient for the stabilisation of chaotic motions. One advantage of this method is that it does not require any analytical knowledge of the system's dynamics. Note that the the two parameters, namely the time-difference[3] τ as well as the multiplier ϱ, are specific for each model. Their values matter for the effective operation of chaos control.

Figure 1 illustrates the whole procedure of time-delayed feedback control.

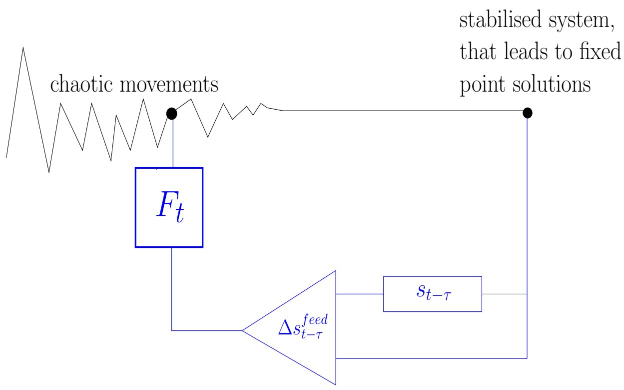

Figure 1: Time-delayed feedback control (own illustration)

[3]Technically, this delay has to coincide with the period of the unstable periodic orbits of the system.

Measuring the output signal at different points in time leads to a feedback. This allows to deduce the intrinsic fluctuation pattern of the chaotic system, which can be then used to generate a control force. Once this is imposed, output oscillations disappear. Hence, we can learn from physicists how chaos can be forced to converge.

3.2 Trend trading as a form of chaos control

Why does this abstract concept of chaos control matter for economists? In order to get to the bottom of this question, I will exploit the knowledge of time-delayed feedback control within the scope of the exchange rate model by De Grauwe and Grimaldi (2006). It also involves dynamic chaos, which converges for certain parameter constellations. Thus, it seems valid to examine whether Pyragas' approach is pertinent and whether it promises new insights concerning the underlying mechanisms of the economic model.

Let us first check applicability of the results following from time-delayed feedback control. The model by De Grauwe and Grimaldi includes different types of agents. One of them are trend-traders.[4] These traders are widely used in the literature and as such they are part of nearly every heterogeneous agent model. In this sense, the model by De Grauwe and Grimaldi is typical for this category of models. Trend traders make their investment decisions according to exchange rate developments in the past. This means that these agents extrapolate past price changes of a currency (Δs_t) into the future. Now let us analyse the concrete rule that trend traders in the model use to forecast fluctuations of the exchange rate. Apparently, they use past time-delayed differences to establish their expectation about future developments:

$$\Delta E_t^c(s_{t+1}) = \beta \sum_{h=1}^{H} \rho_h \Delta s_{t-h}, \tag{3}$$

[4] Note, in the model by De Grauwe/Grimaldi these agents are called "chartists" but for the sake of notational simplicity, I will stick to the more commonly used term of trend traders.

where the elapsed movements of the exchange rate Δs_{t-h} are multiplied by geometrically declining weights ρ_h. These are generated by

$$\rho_h = \frac{(1-\rho)\rho^{h-1}}{(1-\rho^H)},$$

with $1-\rho^H = \sum_{h=1}^{H}(1-\rho)\rho^{h-1}$.
While β applies to the whole sum of past differences, ρ assigns individual weights to them depending on the point in time they occurred. Recent observations are more influential when trend traders form their expectations about the future.

Before analysing the trend traders' expectation rule in its full complexity, let us simplify the equation to the case where they only compare the exchange rate at two different moments in time, i.e. where the number of lags $H = 1$. Then, equation (3) becomes:

$$\Delta E_t^c(s_{t+1})^{H=1} = \beta \Delta s_{t-1}. \tag{4}$$

The extent to which trend traders extrapolate past patterns into the future depends on the coefficient β, which measures the agents' inclination to pay tribute to past changes when forming their expectations about subsequent exchange rates. De Grauwe and Grimaldi define β to be $0 < \beta < 1$. Thus, the expectations of this group will never be completely but always partly determined by past data. This assumption avoids an explosive process.

Compared to the control force in the Pyragas scheme (equation (2)), it can be easily seen that the mechanisms are essentially identical. Both use time-delayed differences $\Delta s_{t-\tau}^{feed}$ and Δs_{t-h} respectively. Furthermore, the two both linearly amplify their feedback variable by a parameter, namely ϱ and the extrapolation parameter β.

Nevertheless, they also differ in some characteristics. First and above all, while only *one* reference point of time is used for stabilisation in the theory of chaos control, trend traders in the general case (equation (3)) consider the exchange rate with regard to *several* points in time. In the simulations published by De Grauwe and Grimaldi trend chasers integrate five lags into their forecasting rules ($H = 5$). Even though the control scheme and the trend traders' extrapolation pattern differ concerning the number of time-delayed differences, this does not distort the general results implied by time-delayed feedback control. Since the publication by Pyragas, the limitation of using one single period has been overcome and control models have been proposed which use multiple delay times.[5]

The second difference concerns the geometrically declining weight ρ_h given to exchange rate movements which have occurred earlier in the past. On this point the economic model simply conducts some internal weighting of the fluctuation patterns in the past. This parameter therefore remains without impact on the general result of stabilisation.

One can therefore state that trend traders, who rely on past movements of the market exchange rate, introduce a time-delayed feedback. This corresponds to Pyragas' control difference. As trend traders make their investment decisions according to their expectations about future market developments, the time-delayed differences are also re-introduced into the model. It is the presence of trend traders' extrapolative forecasting rule, which induces the market exchange rate to stabilise for certain parameter constellations.

4 Conclusion

In this paper I used results from the latest research about chaos control in physics. They help to examine *why* heterogenous agent models which include trend traders generally converge after a certain time. As yet, this property has been described but has remained unexplained. This is quite astonishing as convergence behaviour

[5] Just/Benner/Schöll (2003) provide a comprehensive overview of experimental and theoretical time-delayed feedback control.

may be one of the pieces which lead to a deeper understanding of how the model's outcome is determined.

First, I briefly presented the time-delayed feedback control due to Pyragas. To ease exposition, this method was applied to an economic context. Considering the heterogeneous agent model with trend traders by De Grauwe and Grimaldi, it became apparent *why* the exchange rate stabilises for certain parameter values: the trend traders' forecasting rule, which compares market exchange rates at different moments of time, corresponds to a feedback control mechanism. By this means, trend traders establish a feedback force upon which they build their investment decision. As they behave accordingly, the time-delayed differences are re-introduced into the system and thereby stabilise the model's outcome. That is why, the market exchange rate in the examined fictitious foreign currency market converges after a certain time. This result generalises to all models which include trend traders.

Hence, the concept of time-delayed feedback control in physics indeed provides a better understanding concerning the stability properties of many heterogeneous agent models. My results can therefore be seen as a motivation that future research in this field could lead us to a yet deeper understanding of the true technical and economic forces behind these models' behaviour.

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