Life Expectancy, Health Expenditure and Saving.

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Abstract

The aim of this paper is to investigate the relationship between saving and health expenditure in a two-period overlapping generations economy. Individuals work in the first period and live in retirement in the old age. Health investment is an activity that increases the quality of life and the probability of surviving from the first period to the next. Empirical evidence shows that both health spending and saving, i.e. the consumption when old, appear to be luxury goods but their behavior are strongly different according to the level of per capita GDP. The share of saving on GDP increases proportionally with respect to per capita GDP. On the opposite, the share of health expenditure on GDP increases more than proportionally with respect to per capita GDP. Therefore, the ratio of health investment to saving is nonlinear with respect to per capita GDP, i.e. first increasing and then decreasing. In the proposed model, the ratio of health spending to saving is equal to the ratio between the elasticity of the survival function and the elasticity of the utility function. We prove that the model can replicate empirical results if the utility function is HARA (hyperbolic absolute risk aversion) and the survival function presents a positive and increasing elasticity with respect to health investment. Moreover we show that CES (constant elasticity of substitution) preferences are not consistent with empirical evidence.

Keywords: Intertemporal Choice, Health Spending, Adult Mortality, Saving
JEL Classification: D91, I12, E21

1 Introduction

Through the last two centuries, economic development gradually contributed to the increase in the human life span. In 1840 life expectancy at birth was 40 years in England, 44 years in Denmark and 45 years in Sweden (Livi-Bacci, 1997). According to recent life tables, in 2000 life expectancy at birth in England, Denmark and Sweden is 77, 76 and 79 years respectively. In particular, in most developed countries, life expectancy at birth is around 80 years (World Bank, 2004).

The increase in life expectancy has significant implications for various aspects of the society. In the literature, Bloom et al. (2003), and Kageyama (2003), Zhang et al. (2003), for example, show that increases in life expectancy lead to higher savings rates. This is because the working age increases their saving to finance increased consumption needs in the old age (Modigliani and Brumberg (1980)). Blackburn and Cipriani (2002) analyze the relationship between life expectancy, human capital and fertility.

Other contributions analyze the willingness of people to pay to reduce mortality risk. The willingness to pay criterion, discussed by Schelling (1968) is based on the principle that living is a generally enjoyable activity for which consumers should be willing to sacrifice other pleasures, such as consumption. Murphy and Topel (2003), and Enrlich and Yin (2004) are more recent examples that consider the willingness to pay to reduce mortality risk and an estimate of the value of life. The willingness to pay is determined by the expected discounted present value of lifetime utility and richer societies invest proportionally more in health because life itself is more valuable.

Grossman (1972) develops a model of the demand for the commodity "good health", in which agents demand health since it increases the time available for market and non market activities. Indeed, a rise in the stock of health reduces the amount of time lost for these activities and the monetary value of this reduction is an index of the return to the investment in health (Grossman, 1972). A central result of the Grossman model is that the consumer's demand for health and medical care is positively correlated with his\her wage rate and his\her education level.

Finally Jones and Hall (2006) examine the optimal choice between length of life and consumption. They show that health is a superior good, i.e. as income rises satiation occurs more rapidly in consumption rather than in health spending.

The aim of this paper is to analyze the direct effect of health investment on life expectancy. This framework allows us to investigate the agent's decision on the allocation of total resources between saving and health investment, i.e the consumption in old age and the length of life. We analyze a two-period overlapping generations model in which agents work in the first period and live in retirement in the old age. Health investment is an activity that increases the quality of life and the probability of surviving from the first period of life to the next. Longevity is strictly related to agent's specific health level which in turn offers an important contribution to agent's enjoyment of life (Ehrlich and Chuma, 1990). On the other hand, agent can ensure a good quality of life in the old age by increasing the saving in the working age.

Empirical evidence shows that both health spending and saving, i.e. the consumption when old, appear to be luxury goods but their behaviors are strongly different according to the level of per capita GDP. The share of saving on GDP increases proportionally with respect to per capita GDP. On the opposite, the share of health expenditure on GDP increases more than proportionally with respect to per capita GDP. Therefore, the ratio of saving to health investment is nonlinear with respect to per capita GDP. In particular, this relationship results to be first increasing and then decreasing.

In the proposed model, the ratio of health spending to saving is equal to the ratio between the elasticity of the survival function and the elasticity of the utility function. We prove that the model can replicate empirical results if the utility function is HARA (hyperbolic absolute risk aversion) and the survival function presents a non-constant elasticity with respect to health investment. Moreover we show that CES (constant elasticity of substitution) preferences are not consistent with empirical evidence.

The structure of the paper is outlined as follows. Section 1 presents empirical analysis. Section 2 introduces the general model. Section 3 discusses some possible specification of the instantaneous utility function and the survival function. Section 4 demonstrates that using HARA (hyperbolic absolute risk aversion) preferences we can replicate empirical results. Finally, section 5 draws some concluding remarks.

2 Empirical evidence

The data used in the analysis are taken from World Development Indicators (World Bank, 2004), they are for the period 1960-2002 and cover 208 countries. In Figure 1 we present a recent version of the Preston curve (1975), that is the international relationship between adult survival rate[1] and per capita GDP in purchasing power parity.

Whereas Preston (1975) uses the data on life expectancy we use the data on the survival rate that is less sensitive to child mortality. This is because we are interested in adult's health investment decisions to improve his\her probability of surviving to

[1]The survival rate is the difference between 1 and adult mortality rate. The adult mortality rate is defined from the World Bank as the probability of dying between the ages of 15 and 60, that is, the probability of a 15-year-old dying before reaching age 60, if subject to current age specific mortality rates between ages 15 and 60.

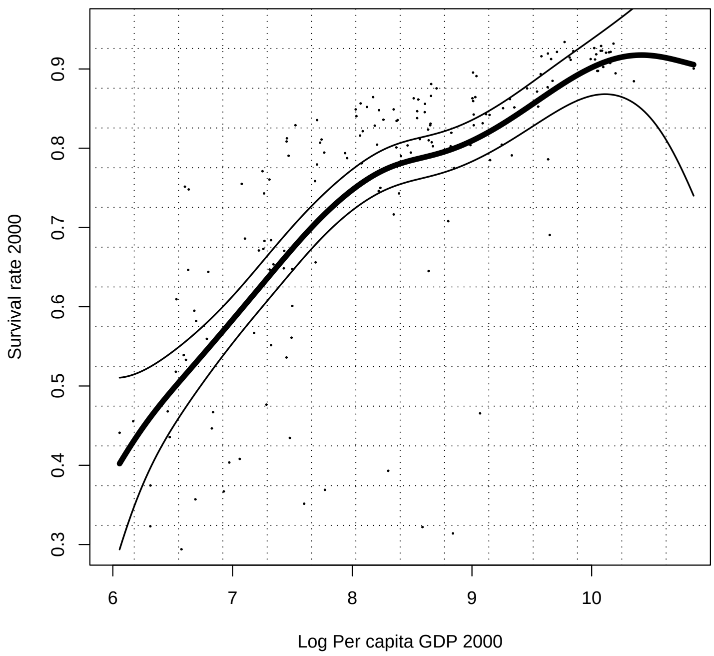

Figure 1: The Preston Curve: Survival Rate versus GDP Per Capita.

old age[2].

We estimate the Preston curve using a cross-country nonparametric regression for the year 2000 and for 164 countries.

We prefer to perform nonparametric regression since it allows us to investigate the relationship between dependent variable and one or more explanatory variables, without making any a priori explicit or implicit assumption about the shape of such relationship.

The cross-country regression in Figure 1 identifies clearly a nonlinear relationship between survival rate and per capita GDP. This can be seen by the wideness of the reference bands. In particular, the width of the reference bands is determined by the standard error of nonparametric regression under the assumption that the linear model holds[3] (Bowman and Azzalini, 1997, Bowman and Azzalini, 2003).

Thus, Figure 1 shows that in low income countries, increases in the per capita GDP

[2]However if we use the data on life expectancy the path of the life expectancy with respect to the per capita income is very similar to the path of the survival rate.

[3]Bowman and Azzalini (1997) discuss the use of reference bands as a graphical display of the level of agreement between a nonparametric curve estimate and a reference model of interest. Particular reference models of interest include no effect, represented by a horizontal line, and a simple linear regression (Bowman and Azzalini, 2003).

are strongly associated with increases in life expectancy, as income per head rises the relationship flattens out. This relationship reflects the influence of a country's own level of income on mortality through such factor as nutrition, education, leisure and health expenditure. With respect the latter factor Figure 2 shows the direct relationship between survival rate and per capita health investment in 2000 for 159 countries. Per capita health investment includes both public and private expenditures on health. It covers the provision of health services (preventive and curative), family planning activities, nutrition activities, and emergency aid designated for health but does not include provision of water and sanitation (World Bank, 2004). Like the Preston's curve the relationship between survival rate and per capita health investment presents a nonlinear path. Countries with low health expenditure tend to gain more in life expectancy than countries starting with high level of health spending.

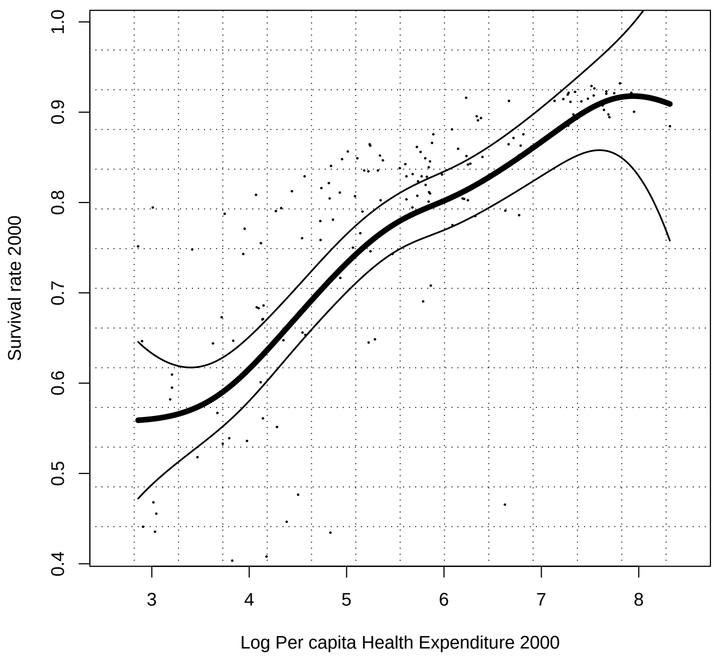

Figure 2: Survival rate versus per capita Health Expenditure.

In figures 3, 4, and 5 we examine the path of health expenditure and saving with respect to income. The aim is to analyze how health increases with respect to different level of income and the relationship between health investment and saving; i.e. investing in health, agents can increase their length of life but they can prefer to devote more resources to the consumption in the old age.

Figures 3 and 4 show respectively nonparametric and semiparametric regressions

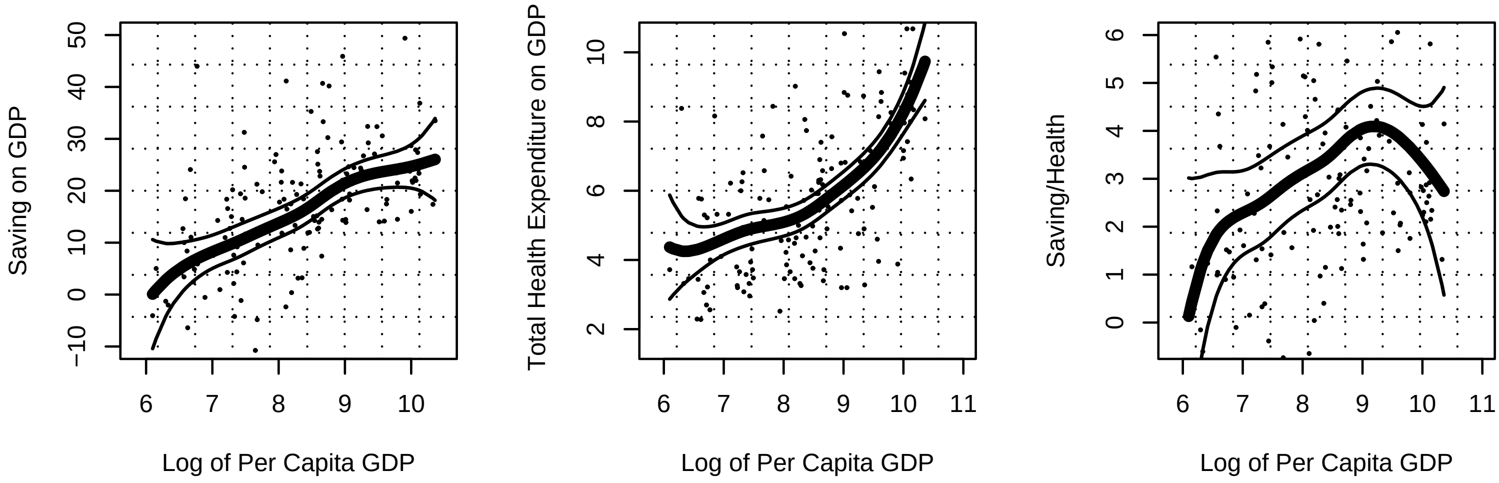

Figure 3: Nonparametric Regression: Saving and Health versus GDP Per Capita

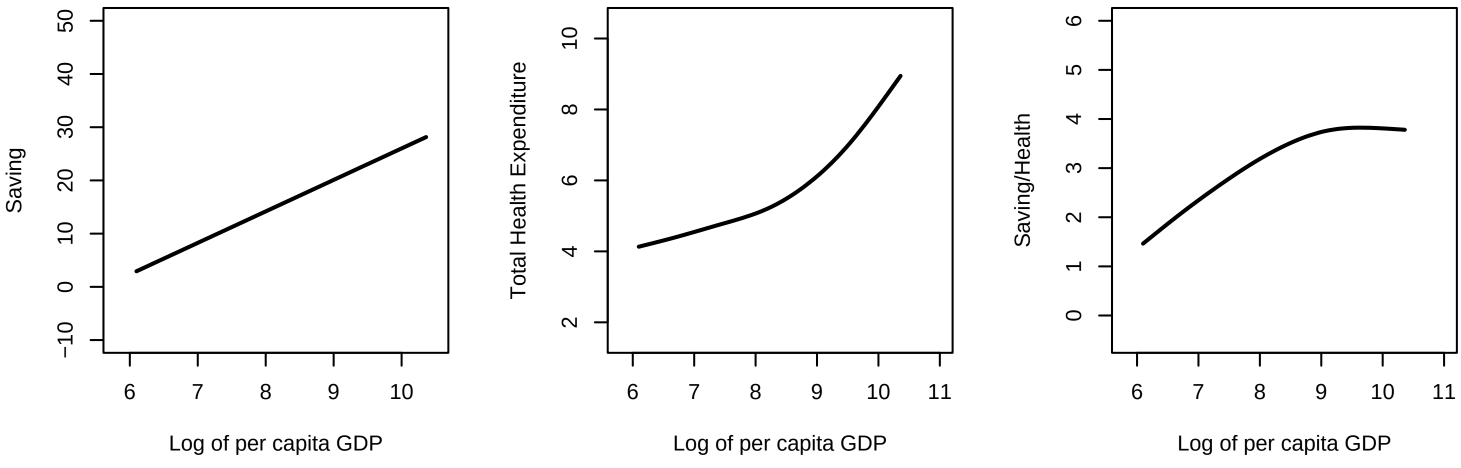

Figure 4: Semiparametric Regression: Saving and Health versus GDP Per Capita .

for the saving on GDP, the health expenditure on GDP and the ratio between saving and health. We consider 147 countries and we make a five years average for the period 1997-2001. The semiparametric regressions is performed using the "mgcv" package available under the R environment. This package provides tools for GAM (generalized additive models) regression. This models combines the simple additive structure of the parametric regression model with the flexibility of the nonparametric approach[4].

Figures 3 and 4 show that both health expenditure on GDP and saving on GDP present a luxury goods behavior. However the path of health share and saving share is strongly different according to different levels of per capita GDP. The share of saving on GDP increases proportionally with respect to per capita GDP whereas the share of health expenditure on GDP increases more than proportionally with respect to per capita GDP. The intuition is that as income increases, the saturation occurs faster in saving than in health spending. In particular, the path of the ratio between saving and health expenditure is nonlinear, it is first increasing and then decreasing. This suggests that the investment in health increases faster than the saving when a country is sufficiently developed.

In the semiparametric regression (figure 4) the nonlinear path of the ratio between saving and health is less evident. This is because semiparametric regressions tend to be more smooth. Table 1 shows the results of the semiparametric regressions. In particular, we estimate the following additive models:

$$m_i = \beta_0 + g(\log GDPPC_i); \tag{1}$$

$$s_i = \delta_0 + h(\log GDPPC_i); \tag{2}$$

$$sm_i = \alpha_0 + f(\log GDPPC_i); \tag{3}$$

where m_i is the health expenditure on GDP, s_i is the saving on GDP, sm_i is the average of the ratio between saving and health expenditure over the five years period, $GDPPC$

[4]In "mgcv" package, smooth terms are represented using penalized regressions spline, that is, the smooth functions are rewritten using a suitable chosen set of basis functions. In particular the model is fitted by minimizing:

$$\sum_{i=1}^{n} \{y_i - f(x_i)\}^2 + \lambda \int [f''(x_i)]^2 \, dx$$

where λ is the smooth term and $\lambda \int [f''(x_i)]^2$ is the roughness penalty term which ensures that the cost of a particular curve is determined not only by its goodness of fit to the data as quantified by the residual sum of squares $\sum_{i=1}^{n} \{y_i - f(x)\}^2$ but also by its roughness $[f''(x_i)]^2$(Green and Silverman, 1994).

The trade-off between model fit and model smoothness is controlled by the smoothing parametric associated to each penalty. If this parameter tend to zero the curve estimated track the data too closely, if tend to infinity the curve will approach to linear regression.

The smoothing parameter is chosen minimizing the generalized cross validation (GCV). In particular the efficient minimization of GCV score is the key to the approach used in "mgcv" package.

is the per capita GDP, g, h and f are unknown smooth functions, and β_0, δ_0 and α_0 are unknown parameters.

	Linear Term			Nonlinear Effects		
	Est.	St.Er.	t	edf	F	p
Eq. (1)						
β_0	5.7595	0.1386	41.56			
$g(\log GDPPC_i)$				2.568	15.12	2.29e-13***
Eq. (2)						
δ_0	16.2233	0.9005	18.02			
$h(\log GDPPC_i)$				1	55.02	9.22e-12***
Eq. (3)						
α_0	3.1947	0.2528	12.64			
$g(\log GDPPC_i)$				1.73	2.814	0.0275*

Table 1: Summary of GAM model (identity link, penalized regressions splines, error Gaussian distribution)

In Table 1 we can see that the "edf"[5] for the relationship between saving and GDP per capita are equal to 1, this implies that this relationship is nearly linear. Indeed performing a linear model we have that the coefficient estimated for the per capita GDP is statistically significant[6].

Figure 5 shows together the path of saving share and health share in nonparametric and semiparametric regressions. We can see that when the log of per capita GDP is very low (6 and 7) the saving share is below the health investment. This result can be explained by the fact that health investment covers a part of public expenditure as emergency aid.

It is possible to give different explanations for the luxury good behavior of health expenditure. One explanation can be the progressiveness of the tax schedule, since the average tax rate increases with income, or an increase in the social demand for the health sector that causes a grows in the government health expenditure in higher proportion than GDP. Others explanations are based on individuals preferences. The idea is that as income grows individual preferences extend not only on the amount of the good consumed but also on the quantity of life. When people became richer

[5]The estimated degrees of freedom ("edf" in table 1) associated with each smooth term are determined entirely by the number of basis functions.

[6]Given the model:

$$s = \hat{\beta} y + \epsilon$$

linear regression yields: $\hat{\beta} = 5.9161$, standard error$= 0.7976$, t value$= 7.418$.

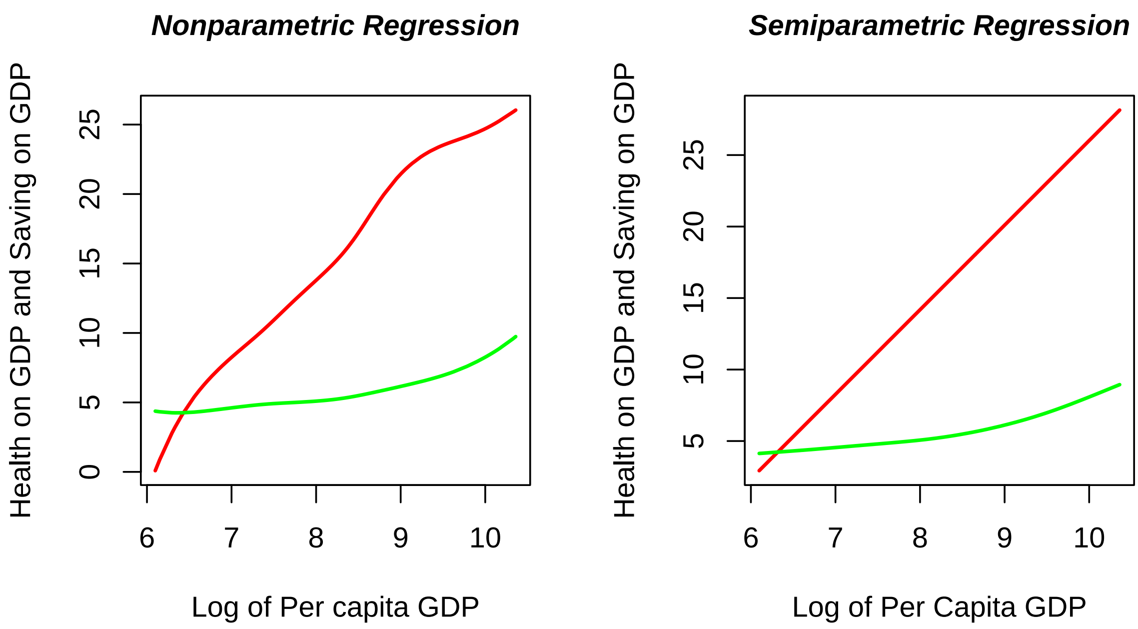

Figure 5: Nonparametric versus Semiparametric Regression.

decide to increase the consumption of health services to extend their life expectancy. An higher longevity allows agents to enjoy additional period of utility (Jones, 2004, Jones and Hall, 2005). In the next section we examine a model based on the latter explanation.

3 The general model

In this section we propose a general model in which the agent's lifetime depends on the length of life and on the health level that directly enters in the utility function.

We consider an overlapping generations economy in which agents live for two periods "youth" and "old age". At the end of the first period agents give birth to a single child. Parents are non altruistic and when they do not survive to the old age, their saving is passed on their offspring as unintended bequest. Thus in the first period of life agents inherit a certain amount of wealth as unintended bequest[7], $b_t \geq 0$, and work

[7]The unintended bequest b_t is given by the saving of the parents that did not survive to the old age, that is:

$$b_t = (1 - p_{t-1})s_{t-1}$$

This implies that in period t agents whose parents die prematurely have higher endowment. In the proposed model we assume that the initial distribution of wealth is given.

receiving a constant wage equal to $\overline{w}$. This amount is allocated between current consumption, health expenditure and saving for the old age consumption . Thus, in the first period, the budget constraint of the representative agent is:

$$c_t = y_t - m_t - s_t, \tag{4}$$

where $y_t = \bar{w} + b_t$ is the agent's endowment, m_t is the health investment[8] and s_t is the saving.

In the second period agents live in retirement and consume entirely their savings, hence the budget constraint in the old age is:

$$c_{t+1} = s_t R, \tag{5}$$

where R is the constant interest rate in the period $t+1$.

Agents have a probability of surviving to the second period which depends on the health investment undertaken in the working age. Following empirical evidence (see Figure 2), the probability of surviving increases with health investment, then:

$$p_t = p(m_t), \tag{6}$$

where $p_t \in (0, \bar{p}]$, $p'_t > 0$, $p''_t < 0$.

Health investment, beyond the increase in the length of life, allows agents to enjoy better life. Thus we suppose that agents in the second period derive utility from their health level (Grossman, 1972)[9] that we specify as:

$$h_{t+1} = h\left(m_t\right) \tag{7}$$

[8]We suppose a perfect substitutability between public health expenditure and private health spending. This implies that a higher proportion of government expenditure devoted to health services reduce private health spending. Indeed, health investment, m_t, in the consumer's budget constraint is the sum of private health investment, m_t^{PRI}, and public health investment, m_t^{PUB}. The latter is equal to a proportional tax on income that is $m_t^{PUB} = \tau y_t$.

The budget constraint is:

$$c_t = (1-\tau)\, y_t - m_t^{PRI} - s_t$$

and substituting $m_t^{PUB} = \tau y_t$ we have that:

$$c_t = y_t - s_t - \left(m_t^{PRI} + m_t^{PUB}\right)$$

where $m_t^{PRI} + m_t^{PUB} = m_t$ in Eq. (4).

The idea is that if agents pay high tax then receive high public health services and therefore decide to devote a low proportion of income to private health expenditure. Otherwise when public health is low private health investment will be very high.

[9]In particular Grossman (1972, 1999) assume that individuals inherit an initial amount of health that depreciates with age and can be increased by investment in health services:

$$h_{t+1} = m_t + (1-\delta_t) h_t$$

For simplicity we consider health level a linear function of health investment, that is:

$$h_{t+1} = m_t \tag{8}$$

The lifetime utility of a representative agent is:

$$U_t = u(c_t) + \beta p(m_t)\hat{u}(c_{t+1}, h_{t+1}) + [1 - p(m_t)]\, M, \tag{9}$$

where $0 < \beta < 1$ is the psychological discount factor, M is the utility in the death state(Rosen, 1988), $u(c_t)$ is the utility in the first period, and $\hat{u}(c_{t+1}, h_{t+1})$ is the utility in the second period which is defined as separable function. In particular, if agents survive to the second period enjoys an utility which depends on consumption and health. For simplicity we assume zero utility from death, $M = 0$[10]. Therefore, substituting Eq.(8) into Eq. (9) we get:

$$U_t = u(c_t) + \beta p(m_t)\hat{u}(c_{t+1}, m_t). \tag{10}$$

3.1 Optimal saving and investment in health

Proposition 1 characterizes the optimal condition for saving and investment in health:

Proposition 1 *The optimal allocation of resources implies that the ratio of saving to health investment is:*

$$\frac{s_t}{m_t} = \frac{\varepsilon_{\hat{u}_c}}{\varepsilon_{\hat{u}_m} + \varepsilon_p}, \tag{11}$$

where $\varepsilon_{\hat{u}_c} = \hat{u}_c(c_{t+1}, m_t)c_{t+1}/\hat{u}(c_{t+1}, m_t)$ *is the elasticity of the instantaneous utility function with respect to consumption,* $\varepsilon_{\hat{u}_m} = \hat{u}_m(c_{t+1}, m_t)m_t/\hat{u}(c_{t+1}, m_t)$ *is the elasticity of the instantaneous utility function with respect to health investment*[11] *and* $\varepsilon_p = p'(m_t)m_t/p(m_t)$ *is the elasticity of survival function with respect to health investment.*

where m_t is the investment in health, δ_t is the depreciation rate that depends on age, and h_t is the inherited health level.

[10]Following Rosen (1998) the expected utility in the second period is:

$$EU = p(m_t)u(c_{t+1}, h_{t+1}) + (1 - p\,(m_t))\, M$$

Subtracting M from utility in each state normalizes the utility of nonsurvival to zero:

$$EU = p(m_t)\,[u(c_{t+1}, h_{t+1}) - M] + (1 - p\,(m_t))\,[M - M]$$

We have that:

$$\tilde{u}(c_{t+1}, h_{t+1}) = u(c_{t+1}, h_{t+1}) - M$$

Therefore is the differences in utility between life and death that matters.

[11]We define:

$$\hat{u}_c = \frac{\partial \hat{u}(c_{t+1}, m_t)}{\partial c_{t+1}},$$

Proof. The first order conditions of expression (10) with respect to s_t and m_t given the constraints (4) and (5) are:

$$\frac{u'(c_t)}{\hat{u}_c(c_{t+1}, m_t)} = \beta p_t(m_t)R, \tag{12}$$

and:

$$u'(c_t) = \beta p_t'(m_t)\hat{u}(c_{t+1}, m_t) + \beta p(m_t)\hat{u}_m(c_{t+1}, m_t). \tag{13}$$

The substitution of Eq. (13) in Eq. (12) yields the ratio between the saving and health investment. ■

Eq. (12) is the usual condition that requires the marginal rate of substitution between current and future consumption should to be equal to the expected return on saving. Eq. (13) captures the trade-off between the marginal cost and marginal benefit of health care spending. By investing in health care, agents renounce to the current consumption for a higher chance of survival in the second period and higher health level in the old age.

According to Proposition 1 the relationship between the saving, health investment and income depends on the behavior of the elasticities in Eq. (11). Empirical evidence (Figures 3, 4 and 5) shows that both saving and health investment rise with income but saturation occurs faster in the saving than in health spending. The intuition is that when income becomes higher than a certain threshold, consumption elasticity falls relative to the health elasticity causing the ratio between saving and health to decrease.

4 Alternative specifications of the Utility function and the Survival function

In this section we analyze how alternative specifications of instantaneous utility function and survival function affects the ratio between saving and health investment in Eq.(11).

4.0.1 Constant elasticity of utility function and survival function

Figure 3 and 4 show that the ratio between saving and health investment rises when per capita GDP is low, and, it is decreasing when per capita GDP is high. In Eq. (11), this empirical paths implies that, when income is low, $\varepsilon_{\hat{u}_c} > \varepsilon_{\hat{u}_m} + \varepsilon_p$, and when income increases, $\varepsilon_{\hat{u}_c} < \varepsilon_{\hat{u}_m} + \varepsilon_p$.

and:

$$\hat{u}_m = \frac{\partial \hat{u}(c_{t+1}, m_t)}{\partial m_t}.$$

The intuition is that when income rises the marginal utility of consumption decreases faster than the marginal utility of health spending. Using an utility function with constant elasticity with respect consumption and health investment, e.g. $\hat{u} = \left[c^{\beta} m^{1-\beta}\right]^{1-\gamma} / (1-\gamma)$, and a survival function with constant elasticity with respect to health investment, i.e. $p = m^{\delta}$, we cannot replicate empirical evidence since s_t/m_t is constant. Indeed, from Eq. (11) we have $s_t/m_t = \beta (1-\gamma) / ((1-\beta)(1-\gamma)+\delta)$.

4.0.2 Constant elasticity of utility function with respect to consumption

Using an utility function with constant $\varepsilon_{\hat{u}_c}$and non-constant $\varepsilon_{\hat{u}_m}$, and a survival function $p(m)$ with non-constant ε_p, we have that the ratio s_t/m_t is consistent with empirical evidence if the sum $\varepsilon_{\hat{u}_m} + \varepsilon_p$ is first decreasing and then increasing. This specification implies that the model is intractable with analytical tools.

4.0.3 Constant elasticity of utility with respect to investment in health

In a model with non-constant $\varepsilon_{\hat{u}_c}$, constant $\varepsilon_{\hat{u}_m}$and non-constant ε_p the path of the ratio s_t/m_t depends on the movements of $\varepsilon_{\hat{u}_c}$, ε_p and on the value of the constant elasticity $\varepsilon_{\hat{u}_m}$.

In the next section we present a model where the utility function presents a zero elasticity with respect to health investment. This specification allows us to replicate the empirical evidence.

5 A Model with zero elasticity of utility with respect to investment in health

In this section we present a simplified version of general utility function displayed in Eq. (9). Health does not enter in the utility function and affects only the survival function. Thus, the lifetime utility takes the following form:

$$U_t = u(c_t) + \beta p(m_t) u(c_{t+1}) \tag{14}$$

subject to the budget constraints given by the Eq. (4) and Eq. (5).

5.1 Survival function

Given Eq. (6) we specify the following probability of surviving to old age:

$$p(m_t) = \begin{cases} \underline{p} + \lambda m_t^{\delta}, & \text{if } m_t \in [0, \hat{m}] \\ \overline{p} & \text{if } m_t > \hat{m} \end{cases} \tag{15}$$

where, $p(0) = \underline{p}$, $0 < \delta < 1$, $\lambda > 0$ and $\overline{p}$ is the highest probability of surviving. This means that an increase in the level of health investment beyond $\hat{m}$ cannot increase the probability of surviving[12]. In particular:

$$\hat{m}_t = \left(\frac{\overline{p} - \underline{p}}{\lambda} \right)^{1/\delta}. \tag{16}$$

The elasticity of the survival function is concave with respect to health investment, that is:

$$\varepsilon_p(m_t) = \frac{\delta \lambda m_t^\delta}{\lambda m_t^\delta + \underline{p}}, \tag{17}$$

where:

$$\varepsilon_p(0) = 0,$$
$$\lim_{m \to \infty} \varepsilon_p = \delta.$$

5.2 Preferences

An instantaneous utility function with an elasticity that depends on consumption is represented by H.A.R.A. (hyperbolic absolute risk aversion function)[13] preferences:

$$u(c) = \frac{(\theta + \sigma c)^{\frac{\sigma-1}{\sigma}}}{\sigma - 1}, \tag{18}$$

where[14] $\theta > 0, \sigma > 1$.

Given Eq. (14) we that Eq. (11) becomes:

$$\frac{s_t}{m_t} = \frac{\varepsilon_{u_c}}{\varepsilon_p}. \tag{19}$$

[12]Empirical analysis (figure 5) shows that in rich countries health investment is still increasing. This stylized fact support the idea that health investment did not yet reach its maximum level $\hat{m}$.

[13]The HARA family is rich, in the sense that by suitable adjustment of the parameters we can have an utility function with absolute o relative risk aversion increasing, decreasing or constant (Merton, 1992). Thus, isolelastic (constant relative risk aversion for $\theta = 0$), exponential (constant absolute risk aversion) and quadratic utility functions are subsets of HARA family. In particular:

$$\begin{aligned} &if \quad \sigma > 0 \Rightarrow D.A.R.A \\ &if \quad \sigma < 0 \Rightarrow I.A.R.A \\ &if \quad \sigma = \infty \Rightarrow A.R.A = 0 \end{aligned}$$

In the paper we assume that $\sigma > 0$.

[14]With this utility function we have that the elasticity of utility increases with consumption. In particular the elasticity is:

$$\varepsilon_{u_c} = \frac{c(\sigma - 1)}{\theta + \sigma c}$$

The ratio between saving and health investment is equal to the ratio of the elasticity of the probability function with respect to health investment and the elasticity of the utility with respect to consumption in the old age.

Given Eq. (15) and (18), Eq. (19) yields the following relationship between the saving and health investment:

$$\frac{s_t}{m_t} = \frac{1}{\delta}\left(\frac{\sigma-1}{\sigma}\right)\left(1+\frac{\underline{p}}{\lambda m_t^{\delta}}\right) - \frac{\theta}{\sigma R m_t}, \tag{20}$$

which implies that the saving is concave in health investment, i.e. $\partial s_t/\partial m_t > 0$ and $\partial^2 s_t/\partial m_t^2 < 0$ (see Appendix C).

The first order conditions corresponding to Eq. (14) in the range $[0, \hat{m}]$ are given by:

$$c_t = \frac{\theta + \sigma c_{t+1}}{\sigma\left[\beta R\left(\underline{p} + \lambda m_t^{\delta}\right)\right]^{\sigma}} - \frac{\theta}{\sigma}, \tag{21}$$

$$c_{t+1} = R\left(\frac{\sigma-1}{\sigma}\right)\frac{m_t}{\delta}\left(1+\frac{\underline{p}}{\lambda m_t^{\delta}}\right) - \frac{\theta}{\sigma}. \tag{22}$$

From Eq. (4), Eq. (22) and Eq.(21) we obtain the the following implicit relation between health investment and income, that is:

$$F(y_t, m_t) = 0$$

where:

$$F(y_t, m_t) \equiv \left(\frac{\sigma-1}{\sigma}\right)\frac{m_t}{\delta}\left(1+\frac{\underline{p}}{\lambda m_t^{\delta}}\right)\left[\frac{R^{1-\sigma}}{\left[\beta\left(\underline{p}+\lambda m_t^{\delta}\right)\right]^{\sigma}} + 1\right] + m_t - y_t - \frac{\theta}{\sigma}\left[1+\frac{1}{R}\right] \tag{23}$$

We are interested in analyzing the behavior of saving and health investment according to different levels of per capita income. The aim is to show that the elasticity of saving falls more rapidly than the elasticity of health investment, that is as people became richer, saving rises but they prefer to devote an increasing share of income to additional years of life.

Proposition 2 *In the range $[0, \hat{m}]$, a sufficient condition to have health investment increasing and convex in income, i.e. $\partial m_t/\partial y_t > 0$ and $\partial^2 m_t/\partial y_t^2 > 0$, is $\delta \leq \frac{1}{\sigma}$. When this condition is satisfied optimal health share presents the following properties*[15] *(see figure 6):*

(1) $\lim\limits_{m \to m_0} \frac{m_t}{y_t} = \infty$

(2) $\lim\limits_{m \to \hat{m}} \frac{m_t}{y_t} = \frac{\hat{m}}{\hat{y}} > 0$

[15] The value m_0 define the value of m_t so that y_t is equal to zero (see appendix A).

(3) $\frac{\partial(m_t/y_t)}{\partial y_t} = 0$ *for* $y_t = y_m$; $\frac{\partial(m_t/y_t)}{\partial y_t} < 0$ *for* $y_t < y_m$; $\frac{\partial(m_t/y_t)}{\partial y_t} > 0$ *for* $y_t > y_m$

Proof. The technical part of this proposition is proved in Appendix B. ■

Proposition 3 *Optimal saving share in income satisfies the following properties (see figure 6):*

(1) $\lim_{m_t \to m_0} \frac{s_t}{y_t} = -\infty$ *if* $m_0 > \theta/\sigma$

(2) $\lim_{m_t \to \hat{m}} \frac{s_t}{y_t} = \frac{\hat{s}}{\hat{y}} > 0$

(3) $\frac{\partial s_t/y_t}{\partial y_t} > 0$ *if* $m_t > \left\{ [R(\sigma-1)]^{\frac{1}{\sigma}} / \beta R - \underline{p}/\lambda \right\}^{1/\delta}$ *and* $m_t > \theta/\sigma\,(1+1/R)$

Proof. See Appendix C ■

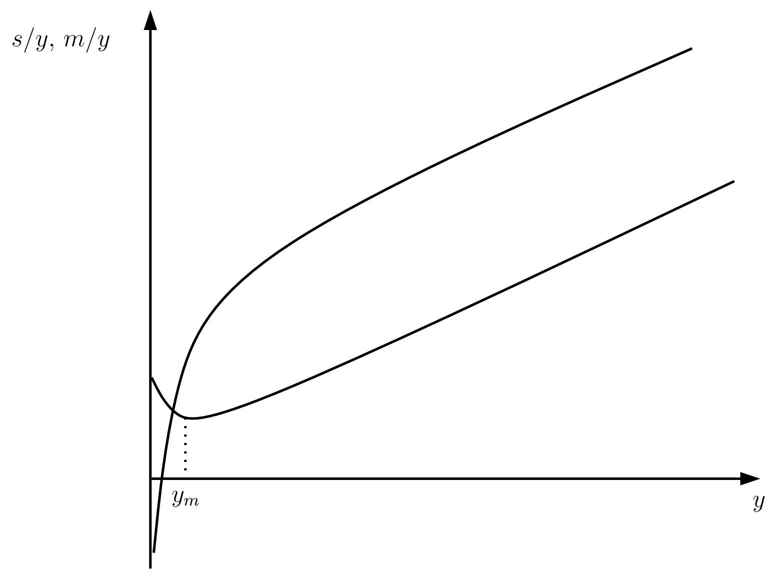

Figure 6: saving share and health share versus income.

Proposition 2 and 3 imply that both saving and health investment behave like luxury goods. In particular, the health share when income is low, i.e. $y_t < y_m$, presents an elasticity with respect to income $\varepsilon_m < 1$. This implies that for low levels of income health is decreasing and the saving rate increases[16] (see figure 6). This indicates that when income is low people does not invest in health and save to finance consumption. When income increases, i.e. $y_t > y_m$, the elasticity of health with respect to income rises, i.e. $\varepsilon_m > 1$.(see Appendix B).

Figure 7 illustrates the results of our calibration for the ratio between optimal saving and optimal health investment with respect to different income levels (our baseline

[16] Our calibration is $\sigma = 2$, $\beta = 0.7$, $R = 2.5$, $\delta = 0.5$, $\theta = 1$, $\lambda = 0.3$, $\underline{p} = 0.2$.

parameters values are $\sigma = 2$, $\beta = 0.7$, $R = 3$, $\delta = 0.5$, $\theta = 1$, $\lambda = 0.3$, $\underline{p} = 0.2$). The following proposition characterizes the properties of the ratio between the saving and income.

Proposition 4 *When $y_t < \tilde{y}$ the saving grows more quickly than health investment; hence the ratio s_t/m_t is increasing as income increases. For $y_t > \tilde{y}$ the ratio between saving and health investment decreases as income increases (see figure 7).*

Proof. See Appendix D ■

Proposition 4 implies that when income is low people devote more resources to the consumption, when income becomes higher than a certain threshold agents spend more income to increase their probability of surviving. Thus for $y_t > \tilde{y}$ while the marginal utility of consumption decreases the marginal utility of additional years of life does not decrease. This implies that as income grows the optimal composition of spending shifts toward health investment (see appendix D).

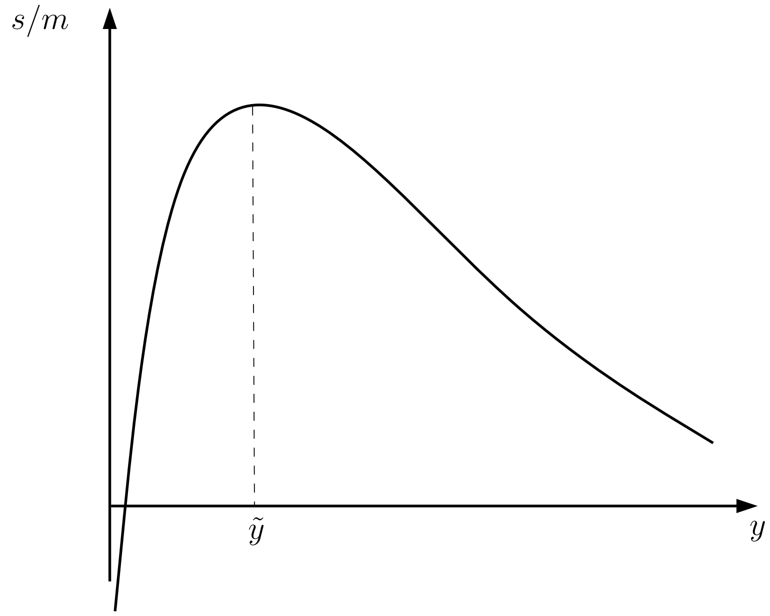

Figure 7: the ratio between saving and health expenditure versus income.

6 Conclusion

This paper analyze agent's decision on the allocation of total resources between health investment and saving. Empirical evidence shows that when income is low agents devote more income to saving to assure consumption in the old age. As income rises the saving continues to rise but health spending increases more quickly. This indicates that for low levels of income, the elasticity of the utility function with respect to consumption is greater than the elasticity of the survival function with respect to health investment. When income rises the opposite occurs. The intuition for this results is that as income grows people become saturated in non-health consumption and choose to spend more income to purchase additional years of life. This mechanism is supported with a theoretical model in which agents present HARA preferences and the survival function shows a non-constant elasticity with respect to health investment.

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Appendix

A Proof of the existence of m_0

When $y_t = 0$, from Eq.(4) we have that:

$$m_t = -\left(c_t + s_t\right),$$

that from Eq. (21) and Eq. (22) yields:

$$m_t + \frac{m_t}{\delta}\left(\frac{\sigma-1}{\sigma}\right)\left(1+\frac{\underline{p}}{\lambda m_t^{\delta}}\right)\left[\frac{R}{\left[\beta R\left(\underline{p}+\lambda m_t^{\delta}\right)\right]^{\sigma}}+1\right]-\frac{\theta}{\sigma}\left[\frac{1}{R}+1\right]=0 \tag{24}$$

We show here the existence of a value of m_t, i.e. m_0, so that the income is equal to zero. The value m_0 can be considered as the activities that agents make to survive when they do not have resources.

From Eq.(24) we can define the two functions:

$$\Phi_1\left(m_t\right)=\frac{m_t}{\delta}\left(\frac{\sigma-1}{\sigma}\right)\left(1+\frac{\underline{p}}{\lambda m_t^{\delta}}\right)\left[\frac{R}{\left[\beta R\left(\underline{p}+\lambda m_t^{\delta}\right)\right]^{\sigma}}\right], \tag{25}$$

$$\Phi_2\left(m_t\right)=\frac{\theta}{\sigma}\left[1+\frac{1}{R}\right]-m_t\left[1+\frac{1}{\delta}\left(\frac{\sigma-1}{\sigma}\right)\left(1+\frac{\underline{p}}{\lambda m_t^{\delta}}\right)\right]. \tag{26}$$

The function in Eq. (25) increases with respect to health investment, that is:

$$\frac{\partial\Phi_1\left(m_t\right)}{\partial m_t}=\left(\frac{\sigma-1}{\sigma}\right)\frac{R}{\left(\beta R\right)^{\sigma}}\left[\frac{\left(1-\sigma\delta\right)\lambda m_t^{\delta}+\left(1-\delta\right)\underline{p}}{\lambda m_t^{\delta}\left(\lambda m_t^{\delta}+\underline{p}\right)^{\sigma}}\right]>0,$$

since $1-\sigma\delta$ is assumed positive from proposition 2, and $\Phi_1\left(0\right)=0$.

The function $\Phi_2\left(m_t\right)$ in Eq. (26) is decreasing with respect to health investment, that is:

$$\frac{\partial\Phi_2\left(m_t\right)}{\partial m_t}=-\left[\frac{\left(\sigma-1+\delta\sigma\right)\lambda m_t^{\delta}+\left(\sigma-1\right)\left(1-\delta\right)\underline{p}}{\delta\lambda m_t^{\delta}}\right]<0$$

and $\Phi_2\left(0\right)=\frac{\theta}{\sigma}\left[1+\frac{1}{R}\right]$.

Thus since the function $\Phi_1\left(m_t\right)$ is increasing in health, the function $\Phi_2\left(m_t\right)$ is decreasing in health, and for $m_t=0$ the function $\Phi_1=0$ and $\Phi_2=\left(\theta/\sigma\right)\left[1+\left(1/R\right)\right]$, then there exist a value of m_t, i.e m_0, such that the two functions intersect.

B Proof of proposition 2

Equation (23) implicitly defines optimal health investment as a function of income. Applying the implicit function theorem to Eq. (23) we get:

$$\frac{\partial m_t}{\partial y_t} = \frac{\sigma\delta\lambda m_t^\delta G\left(m_t\right)}{\left(\sigma-1\right)\left(1-\delta\right)\underline{p}\left[G\left(m_t\right)+R\right]+\lambda m_t^\delta\left[R(1-\sigma\delta)(\sigma-1)+\left(\sigma-1+\sigma\delta\right)G\left(m_t\right)\right]}, \tag{27}$$

where:

$$G\left(m_t\right) = \left[\beta R(\underline{p}+\lambda m_t^\delta)\right]^\sigma$$

A sufficient condition to have health increasing in income is that:

$$\delta \leq \frac{1}{\sigma}. \tag{28}$$

We have that $\partial^2 m_t/\partial y_t^2 > 0$ if:

$$R\sigma\lambda m_t^\delta\left[\sigma\lambda m_t^\delta\left(1-\sigma\delta\right)+\underline{p}\left(1-\delta\right)\right]+\underline{p}\left(1-\delta\right)\left(\lambda m_t^\delta+\underline{p}\right)\left[G\left(m_t\right)+R\right] > 0.$$

which is satisfied when inequality (28) holds.

Analysis of Health Share

Given Eq. (23) the health share is given by the following expression:

$$\frac{m_t}{y_t} = m_t\left\{\left(\frac{\sigma-1}{\sigma}\right)\frac{m_t}{\delta}\left(1+\frac{\underline{p}}{\lambda m_t^\delta}\right)\left[\frac{R^{1-\sigma}}{\left[\beta\left(\underline{p}+\lambda m_t^\delta\right)\right]^\sigma}+1\right]+m_t-\frac{\theta}{\sigma}\left[1+\frac{1}{R}\right]_t\right\}^{-1}, \tag{29}$$

from which we get:

$$\lim_{m\to m_0}\frac{m_t}{y_t} = \frac{m_0}{0} = \infty, \tag{30}$$

and when health investment tend to $\hat{m}$ health share is equal to a positive constant:

$$\lim_{m\to\hat{m}}\frac{m_t}{y_t} = \frac{\hat{m}}{\hat{y}} > 0.$$

Deriving Eq. (29) with respect to income we obtain:

$$\frac{\partial\left(m_t/y_t\right)}{\partial y_t} = \frac{\left(\partial m_t/\partial y_t\right)y_t - m_t}{y_t^2}, \tag{31}$$

where $\partial\left(m_t/y_t\right)/\partial y_t > 0$ implies that:

$$\varepsilon_m = \frac{\left(\partial m_t/\partial y_t\right)y_t}{m_t} > 1, \tag{32}$$

where ε_m is the elasticity of health spending with respect to income. Thus health share increases with income if $\varepsilon_m > 1$, which implies that health investment behaves like a luxury good.

Eq. (31) is given by the following expression:

$$\frac{\partial \left(m_t/y_t\right)}{\partial y_t} = \left(\sigma-1\right) m_t \left(\sigma + dm_t^{\delta}\right) R^2 + \left(\beta p R\right)^{\sigma} \left(R\left(\sigma-1\right) dm_t^{1+\delta} + \theta\left(1+R\right)\right),$$

from which $\varepsilon_m = 1$ if:

$$\frac{\left(\sigma-1\right) m_t \left(\sigma + dm_t^{\delta}\right) R^2}{\left(\beta p R\right)^{\sigma}} = -R\left(\sigma-1\right) dm_t^{1+\delta} + \theta\left(1+R\right). \tag{33}$$

Thus we can analyze the two functions:

$$\psi_1\left(m_t\right) = \frac{\left(\sigma-1\right) m_t^{1-\delta}\left(\sigma\lambda m_t^{\delta} + \underline{p}\right) R^2}{G\left(m_t\right)},$$

$$\psi_2\left(m_t\right) = -\lambda m_t^{1-\delta}\left(\sigma-1\right) R\underline{p} + \theta\left(1+R\right).$$

From condition in Eq. (28) we have that the function $\psi_1\left(m_t\right)$ is increasing in health investment, that is:

$$\frac{\partial \psi_1}{\partial m_t} = \frac{\beta\left(\sigma-1\right) R^3\left[(1-\sigma\delta)\lambda^2 m_t^{2\delta} + \underline{p}\left(1-\delta\right)\left(\sigma+1\right)\lambda m_t^{\delta} + \left(1-\delta\right)\underline{p}^2\right]}{\left(\lambda m_t^{\delta}\right) G\left(m_t\right)^{1+1/b}} > 0$$

and:

$$\psi_1\left(0\right) = 0$$

$$\lim_{m\to\infty} \psi_1\left(m_t\right) = \infty$$

The function ψ_2 decreases in health investment, that is:

$$\frac{\partial \psi_2}{\partial m_t} = -\frac{\left(\sigma-1\right)\left(1-\delta\right) R\underline{p}}{m_t^{\delta}} < 0$$

and:

$$\psi_2\left(0\right) = \theta\left(1+R\right)$$

$$\lim_{m\to\infty} \psi_2\left(m_t\right) = -\infty$$

Thus two function intersect in $\bar{m}$ where $\varepsilon_m = 1$. Substituting this value $\bar{m}$ to the Eq. (23) we obtain the value y_m so that $\varepsilon_m = 1$. When $y_t < y_m$ then $\psi_2\left(m_t\right) > \psi_1\left(m_t\right)$,that is $\varepsilon_m < 1$ and the health share is decreasing in income. When $y_t > y_m$ then $\psi_2(m_t) > \psi_1(m_t)$ and $\varepsilon_m > 1$, that is the health share increases.

C Proof of proposition 3

The relationship between saving and health is positive and concave. That is, differentiation of Eq. (22) with respect to health investment give us:

$$\frac{\partial s_t}{\partial m_t} = \left(\frac{\sigma-1}{\sigma}\right)\frac{1}{\lambda m_t^{\delta}\delta}\left[\underline{p}\left(1-\delta\right)+\lambda m_t^{\delta}\right] \tag{34}$$

and:

$$\frac{\partial^2 s_t}{\partial m_t^2} = -\frac{\left(1-\delta\right)\left(\sigma-1\right)\underline{p}}{\sigma m_t^{(\delta-1)}}$$

where $\partial s_t/\partial m_t > 0$ and $\partial^2 s_t/\partial m_t^2 < 0$.

Deriving the saving with respect to income we obtain that the saving increases with income. In particular from condition in Eq.(28) we obtain that the saving behaves like a luxury good, that is:

$$\frac{\partial s_t}{\partial y_t} = \frac{\partial s_t}{\partial m_t}\frac{\partial m_t}{\partial y_t} > 0$$

Analysis of Saving Share

Eq. (22) and Eq. (23) yield the following expression for the saving share on income:

$$\frac{s_t}{y_t} = \frac{1}{y_t}\left[\left(\frac{\sigma-1}{\sigma}\right)\frac{m_t}{\delta}\left(1+\frac{\underline{p}}{\lambda m_t^{\delta}}\right)-\frac{\theta}{\sigma R}\right]. \tag{35}$$

From Eq. (23) when $y_t = 0$ then:

$$\left(\frac{\sigma-1}{\sigma}\right)\frac{m_t}{\delta}\left(1+\frac{\underline{p}}{\lambda m_t^{\delta}}\right) = \frac{G(m_t)\left[\frac{\theta}{\sigma}\left(1+\frac{1}{R}\right)-m_t\right]}{(R+G(m_t))},$$

from which we obtain that:

$$\lim_{m\to m_0}\frac{s_t}{y_t} = -\infty, \tag{36}$$

if the following sufficient condition is satisfied:

$$m_0 > \frac{\theta}{\sigma}.$$

Moreover:

$$\lim_{m\to\hat{m}}\frac{s_t}{y_t} = \frac{\hat{s}}{\hat{y}} > 0$$

Deriving the saving share with respect to income we get:

$$\frac{\partial(s_t/y_t)}{\partial y_t} = \frac{1}{y_t^2}\left[\frac{\partial s_t}{\partial y_t}y_t - s_t\right], \tag{37}$$

that is:

$$\frac{\partial(s_t/y_t)}{\partial y_t} = \varepsilon_s - 1,$$

where ε_s is the elasticity of saving with respect to income. Thus if $\varepsilon_s > 1$ then saving share is increasing in income.

Eq. (37) is given by the following expression:

$$\left(\frac{\sigma-1}{\sigma}\right)\frac{m_t}{\delta}\left(1+\frac{\underline{p}}{\lambda m_t^{\delta}}\right)\left[\frac{\partial s_t}{\partial y_t}\left(\frac{R}{G(m_t)}+1\right)-1\right]+\frac{\partial s_t}{\partial y_t}\left[m_t-\frac{\theta}{\sigma}\left(1+\frac{1}{R}\right)\right]+\frac{a}{bR}$$

Thus $\partial(s_t/y_t)/\partial y_t > 0$, i.e. $\varepsilon_s > 1$, if the following two sufficient conditions are satisfied:

$$m_t > \left[\frac{[R(\sigma-1)]^{\frac{1}{\sigma}}}{\beta R} - \frac{\underline{p}}{\lambda}\right]^{1/\delta}$$

and:

$$m_t > \frac{\theta}{\sigma}\left(1+\frac{1}{R}\right)$$

D Analysis of the ratio between saving and health

Given Eq. (22) we get that:

$$\frac{\partial(s_t/m_t)}{\partial y_t} = \frac{1}{y_t^2}\frac{\partial m_t}{\partial y_t}\left[\frac{\partial s_t}{\partial m_t}m_t - s_t\right]$$

where from Eq. (28) $\partial m_t/\partial y_t > 0$. From Eq. (34) and Eq. (22) we obtain:

$$\frac{\partial s_t}{\partial m_t}m_t - s_t = \frac{\theta}{\sigma R} - \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\underline{p}}{\lambda}m_t^{1-\delta}\right) = 0,$$

that implies:

$$\tilde{m} = \left[\frac{\lambda\theta}{\underline{p}R(\sigma-1)}\right]^{\frac{1}{1-\delta}}. \tag{38}$$

Substituting Eq. (38) in Eq. (23) we get:

$$\tilde{y} = y(\tilde{m})$$

We have that for $y_t < \tilde{y}$, $\partial(s_t/m_t)/\partial y_t > 0$ that is:

$$\frac{\theta}{\sigma R} - \left(\frac{\sigma-1}{\sigma}\right)\left[\frac{\underline{p}}{\lambda}m_t^{1-\delta}\right] > 0$$

when:

$$m_t < \tilde{m}$$

and for $y_t > \tilde{y}$, $\partial(s_t(m_t)/m_t)/\partial y_t < 0$, that is:

$$\frac{\theta}{\sigma R} - \left(\frac{\sigma - 1}{\sigma}\right)\left[\frac{p}{\lambda} m_t^{1-\delta}\right] < 0$$

if:

$$m_t > \tilde{m}$$

Thus the ratio s_t/m_t for $m_t < \tilde{m}$ is increasing in income and for $m_t > \tilde{m}$ the ratio s_t/m_t is decreasing in income.