

# An Investigation on the Usefulness of SVARs with Long-Run Restrictions

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## 1 Introduction

The main focus in this work is to study whether structural vector autoregression (SVAR) with long-run restrictions is useful as an empirical macroeconomic tool. This is a topic that has been widely discussed in the literature<sup>1</sup>. This paper investigates the pros and cons of SVARs with long-run restrictions by examining three seminal SVAR studies : Blanchard and Quah (1989; henceforth BQ), King, Plosser, Stock and Watson (1991; henceforth KPSW), and Gali (1999). Particular concern is given to the potential of SVARs in grasping important features of the real world and the robustness of the findings. To investigate if the structural VAR approach can be considered as a useful device to recover structures behind the data, the results of seminal SVAR studies are compared in different aspects. In particular, the identified macroeconomic shocks, and the corresponding dynamic responses and business cycle components of the variables in the VARs are considered. The main conclusion is that SVAR can be a useful device for evaluating macroeconomic theories. However, care must be taken in particular when quantifying the prior beliefs about the macroeconomy.

The analysis made in this paper provides a different way for assessing the usefulness of SVARs than the simulation studies. The current assessment is based on real, not artificial data and considers therefore aspects of model building that cannot be checked in a simulation study. Alexius and Carlsson (2005, 2001) study to what extent structural VAR models are capable of capturing the phenomena that they are supposed to capture. They basically compare the identified technology shocks of Gali and KPSW models with different technology shock measures that are computed using the production function approach: The first measure is the conventional Solow residual and the second is the refined Solow residual that is considered by Basu and Kimball (1997) *inter alia*. Their results and approach are related to our paper. The difference of this paper to Alexius and Carlsson (2005, 2001) is that this paper compares only SVAR results. Moreover, the topic of their investigation is only the technology shock. We, on the other hand, consider other macroeconomic shocks and the dynamic responses to the macroeconomic shocks as well.

It is known that model selection is an important step in VAR analysis and the results can be sensitive to the specification of the model. One aspect of this study is to investigate the robustness of SVAR results within individual models. If the robustness lacks to an important degree, the

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<sup>1</sup>See Lippi and Reichlin (1993), Faust and Leeper (1997), Cooley and Dwyer (1998), Breitung (2000), Erceg et al. (2004) and Christiano et al. (2006) among others.

researcher must be more careful when specifying the model and drawing conclusions from it. The robustness check is conducted with respect to the choice of the model variables, the choice of an appropriate variable transformation, the selection of the lag order and the selection of the cointegration rank.

Omitted variables bias is a problem that needs to be paid particular attention. If some variables that are important for the explanation of the specific macroeconomic issue are left out, the identification scheme cannot be credible<sup>2</sup>. We find out that omitted variable bias is not a big problem for the bivariate Gali specification, but for the KPSW models. Our results emphasize the importance of including a labor input variable in SVAR models that study the role of technology shocks in business cycle fluctuations. Excluding this variable results in substantial bias and the SVAR model cannot be informative any more.

Alexius and Carlsson report that identified technology shocks are not much sensitive to different lag orders in the VAR or cointegration space. Our findings confirm their conclusion. Nevertheless, the dynamic multipliers of the structural shocks may be affected seriously. Altogether, the results in our paper turn out to be sensitive to model specification in some, but not all cases. This underlies the importance of considering many possibilities when presenting SVAR results and setting sound rules for model selection.

We make comparisons across the three SVAR models. As noted by Faust and Leeper (1997), checking for consistency of results across various small SVAR models can help to maintain the convenience of small models. According to Blanchard and Quah (1993), *the entire point of structural VAR analysis is exactly identification of alternative disturbances. Thus, all possibilities need to be considered.* In this paper, we show that exploiting the findings from different SVAR studies in a combined way improves the understanding of the role of macroeconomic mechanisms. For example, the technology shock of the three-variable KPSW model, the real interest shock of the six-variable KPSW model and the nontechnology shock of the bivariate Gali model all seem to refer to a similar phenomenon. The researcher faces then the problem of how to call the identified mechanism: Technology, nontechnology, or real interest? It is argued in this paper that the underlying mechanism behind this phenomenon is a nontechnology mechanism. More important is, however, the finding

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<sup>2</sup>Two recently developed VAR approaches called factor-augmented VAR (FAVAR) and global VAR (GVAR) try to remedy the omitted variables bias problem and provide new perspectives also for structural analysis. The reader can refer to Bernanke et al. (2005) and Pesaran et al. (2004) for information on the FAVAR and GVAR approaches, respectively.

that the researcher can collect information about the macroeconomic mechanisms by comparing the results of different SVAR models that do not necessarily contain exactly the same variables.

The next section briefly reviews the two identification schemes considered throughout this paper. Section 3 presents the findings. Section 4 concludes.

## 2 SVARs with Long-Run Restrictions

This section provides a review of two SVAR identification schemes where long-run restrictions are used. The first scheme follows from BQ for systems without cointegration and is also applied by Gali. The second scheme is implemented in KPSW and is for systems with cointegration. Since the techniques are meanwhile well-known, only brief descriptions of them are given here. The reader may refer to Lütkepohl (2006) for more detailed descriptions.

### 2.1 The BQ Identification Scheme

Assume that a vector of  $K$  variables  $Y_t$  where at least the first variable is integrated of order one can be expressed as a distributed lag of white noise residuals  $u_t$  with  $E(L)$  being the lag polynomial containing the moving-average coefficient matrices and  $L$  the lag operator.<sup>3</sup>:

$$Y_t = E(L) u_t. \quad (1)$$

This is the so-called reduced form equation which be written in structural form as

$$Y_t = C(L) \varepsilon_t \quad (2)$$

with  $C(L) := E(L) C$  and  $\varepsilon_t := C^{-1} u_t$ . There are infinitely many  $C$  matrices of order  $K \times K$ . In order to obtain a unique economically meaningful structure from the reduced form equation (1), BQ and Gali impose the restriction that only the first structural shock in  $\varepsilon_t$  can have an effect in the long-run on the first variable in  $Y_t$ <sup>4</sup>. Formally, the first element in  $Y_t$  can be written as

$$Y_{1,t} = \begin{bmatrix} C^{11}(L) & C^{12}(L) & \dots & C^{1K}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \\ \vdots \\ \varepsilon_t^K \end{bmatrix}. \quad (3)$$

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<sup>3</sup>The deterministic terms in the estimated VAR are ignored for the ease of presentation in the following.

<sup>4</sup>Moreover, the conventional atheoretical restriction that the covariance matrix of the structural shocks is an identity matrix is imposed.

The identification restriction mentioned above corresponds to  $C^{12}(1) = \dots = C^{1K}(1) = 0$  with  $C^{ij}(L)$  being the element of  $C(L)$  corresponding to the  $i$ -th variable and  $j$ -th structural shock<sup>5</sup>. The matrix of long-run multipliers  $C(1)$  is assumed to be lower triangular in the BQ identification scheme.

## 2.2 The KPSW Identification Scheme

The identification scheme developed by KPSW is applied to VAR systems with cointegrated variables<sup>6</sup>. The matrix of long-run multipliers can be written as

$$\Gamma(1) = \begin{bmatrix} A & 0 \end{bmatrix} \quad (4)$$

where  $\Gamma(1)$  is the structural matrix of long-run multipliers<sup>7</sup>,  $A$  is of order  $K \times k$  and  $0$  is the zeros matrix of order  $K \times r$ ,  $k$  being the number of stochastic trends and  $r$  the cointegration rank. Furthermore, the matrix  $A$  is assumed to have the form

$$A = \hat{A}\Pi \quad (5)$$

where  $\hat{A}$  is a known  $K \times k$  matrix and  $\Pi$  is a  $k \times k$  lower triangular matrix with ones in its diagonal.

KPSW construct the  $\hat{A}$  matrix by using the parameters of the cointegrating equations which are estimated with the dynamic OLS technique. As Jang and Ogaki (2001) show,  $\hat{A}$  can be determined by exploiting the fact that  $\beta'\hat{A} = 0$  and  $\beta'\beta_{\perp} = 0$  must be valid, where  $\beta$  is the  $K \times r$  matrix of cointegrating vectors and  $\beta_{\perp}$  is its orthogonal with the order  $K \times k$ . Thus, one can choose

$$\hat{A} = \beta_{\perp}. \quad (6)$$

This means that where the cointegrating vectors are estimated there are some free parameters in the matrix  $\hat{A}$  which are not restricted other than  $\beta'\beta_{\perp} = 0$ . Notice that the number of unrestricted parameters in  $\beta_{\perp}$  can be  $r$  times  $k$ , i.e., the cointegration rank times the number of stochastic trends in the VAR.

## 3 The Assessment of the Selected Literature

Gali, BQ and KPSW papers are considered in the next three subsections, respectively. In Subsection 3.4, comparisons across the SVAR models are made. Finally, the findings are discussed further in

<sup>5</sup>Note that the nonstationary variables are first-differenced in this VAR.

<sup>6</sup>The notation in Jang and Ogaki (2001) is used here while describing the KPSW identification scheme.

<sup>7</sup> $\Gamma(1)$  is analogous to  $C(1)$ .

Subsection 3.5. Original data is used when generating the results for Gali and KPSW models<sup>8</sup>. The unemployment rate data is obtained from the Bureau of Labor Statistics (BLS). The sample periods of the original papers are used for the results in Subsections 3.1, 3.2 and 3.3<sup>9</sup>.

A one-standard-deviation structural shock is always considered when plotting the dynamic responses. The business cycle component w.r.t. a certain structural shock means that all the other shocks are set to zero in the SVAR when the historical time series are computed and the result is HP-filtered like Gali does for his analysis.

### 3.1 Gali (1999)

The main model in Gali (1999) is a bivariate VAR containing labor productivity,  $x$ , and labor input,  $n$ . Both variables are nonstationary according to the Augmented Dickey-Fuller (ADF) test, but not cointegrated. The crucial identifying restriction is that the technology shock is the only structural shock that has an impact on labor productivity in the long-run. The sensitivity of the results to some model selection criteria is discussed in the following.

**The selection of the lag order** Gali estimates a VAR with four lags. However, he does not indicate why this lag number is chosen<sup>10</sup>. The first eight observations in the sample are left out to be used as initial values for the results presented here. The correlations reported in Table 1 show that the identified technology and nontechnology shocks are not affected much by the lag order choice. The same cannot be said, however, for the technology and nontechnology components of the variables illustrated in Figure 1. The standard deviations of the technology components of output and labor input are obviously higher when a VAR with eight lags is estimated. It can be concluded from this picture that the dynamic multipliers of output and labor input are sensitive to the lag order choice, but not the identified shocks.

**The selection of model variables** The so-called omitted variables bias can be a relevant issue in particular for bivariate SVAR models with long-run restrictions. If some variables that are important for the explanation of the particular macroeconomic issue investigated are left out, the

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<sup>8</sup>I thank Professor Gali for providing me with his original data. The original KPSW data can be found on the file <http://www.wws.princeton.edu/~mwatson/ddisk/kpsw.zip> provided by Professor Watson.

<sup>9</sup>The sample periods are 1949:1-1988:4 for the three-variable KPSW model, 1954:1-1988:4 for the six variable KPSW model, 1948:1-1994:4 for the bivariate Gali model, and 1948:1-1987:4 for the BQ model.

<sup>10</sup>Different lag selection criteria point to two lags in the bivariate model.

Table 1: Correlations among Identified Shocks in Bivariate Gali Model

	tech,2	tech,4	tech,8	nontech,2	nontech,4	nontech,8
tech,2	1					
tech,4	.98	1				
tech,8	.96	.96	1			
nontech,2	-.00	.14	.01	1		
nontech,4	-.13	.00	-.12	.97	1	
nontech,8	.01	.14	.00	.93	.94	1

Notes: Table 1 shows the correlations among the identified shocks in bivariate Gali models where the lag order is 2, 4, and 8, respectively. tech: technology. nontech: nontechnology.

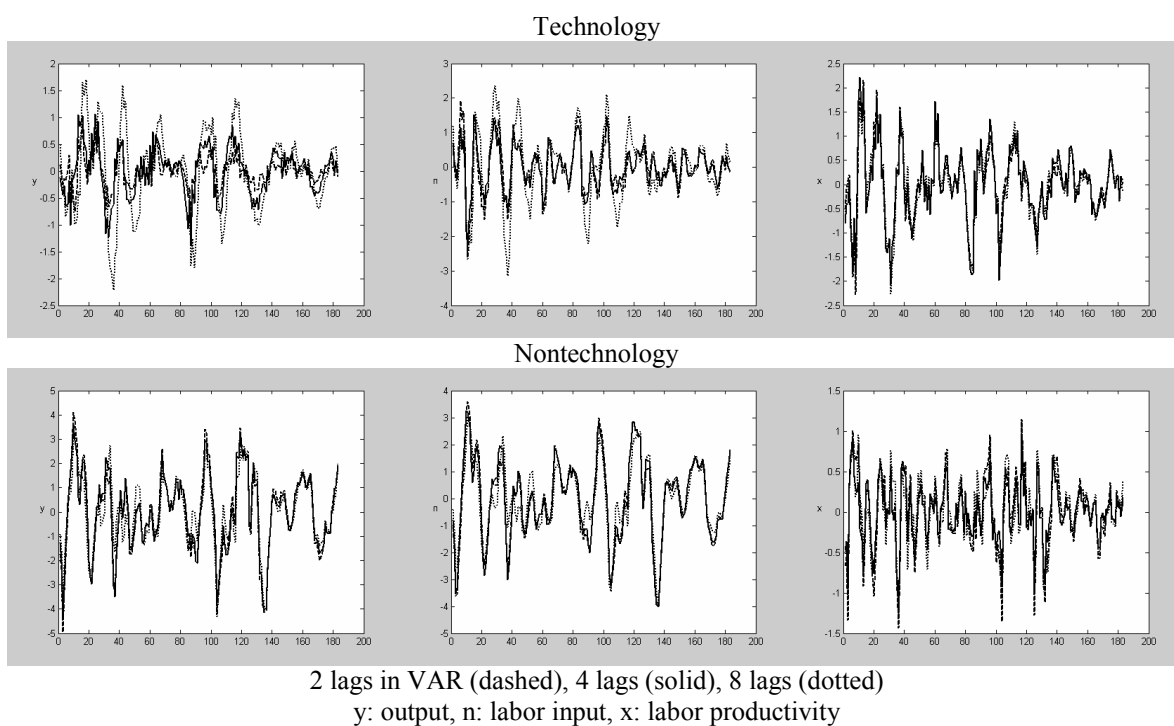


Figure 1: Technology and Nontechnology Components of the Variables in the Bivariate Gali Model

identification scheme may not even be approximately correct. Gali (1999) augments his bivariate model with three more variables, namely real balances,  $m - p$ , real interest rate,  $R - \Delta p$ , and inflation  $\Delta p$ . Inflation and real balances enter the augmented VAR in first difference, and real interest rate in level. Gali illustrates the responses to a technology shock in the five-variable model in Figure 4 of his paper and points to similarities to the bivariate case. A comparison<sup>11</sup> of the identified technology<sup>12</sup> shocks, dynamic responses to a technology shock and technology component of the model variables shows that omitted variables bias is not an important issue for the bivariate model, i.e., the bivariate model seems to be a good approximation for the research question posed in Gali (1999): Do technology shocks explain aggregate fluctuations? The identified technology shocks in bivariate and five-variable models are correlated with a coefficient of 0.92. The output technology cycles are correlated with a coefficient of 0.66 and the labor input technology cycles with a coefficient of 0.78. The correlation coefficients of the nontechnology business cycle components of output, labor productivity and labor input are 0.97, 0.96 and 0.92, respectively<sup>13</sup>.

Figure 2 illustrates the responses of output, labor input and labor productivity in bivariate and five-variable models. We notice that augmenting the model with three additional variables does not create a difference in the short-run, namely one-year, dynamic responses of output and labor input, however in the mid- and long-run it does<sup>14</sup>. The qualitative patterns of the dynamic responses of these variables do not change though. Nevertheless, the long-run effect of a technology shock on labor input becomes positive when the bivariate model is augmented. Overall, the critique of Faust and Leeper (1997) applies to output and labor input, but not to labor productivity<sup>15</sup>. That is, we would expect small changes in the model parameters to cause big changes in the long-run effects of shocks, but this does not happen to be the case for labor productivity as Figures 1 and 2 show.

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<sup>11</sup>The sample period for the computations of this section is 1959:1 - 1994:4 due to the lack of data for M2 before 1959:1 in the sample used by Gali (1999).

<sup>12</sup>Only the technology shocks and their components are considered. Note that the five-variable model does not identify the four nontechnology shocks separately. So, it does not make sense to compare the demand shocks of the bivariate model with the unidentified demand shocks of the five-variable model.

<sup>13</sup>The four nontechnology components of a certain variable are added up for computing the nontechnology business cycle component of it in the five-variable model.

<sup>14</sup>This is basically the reason behind the relatively lower correlations of output and labor input technology cycles in comparison to labor productivity technology cycles.

<sup>15</sup>Notice that it seems as if the dynamic responses of labor productivity are sensitive in the short-run in Figure 2, but this is not so. The scale of the  $y$  axis in the graph of the labor productivity response is just too small.



### The Response to a Technology Shock

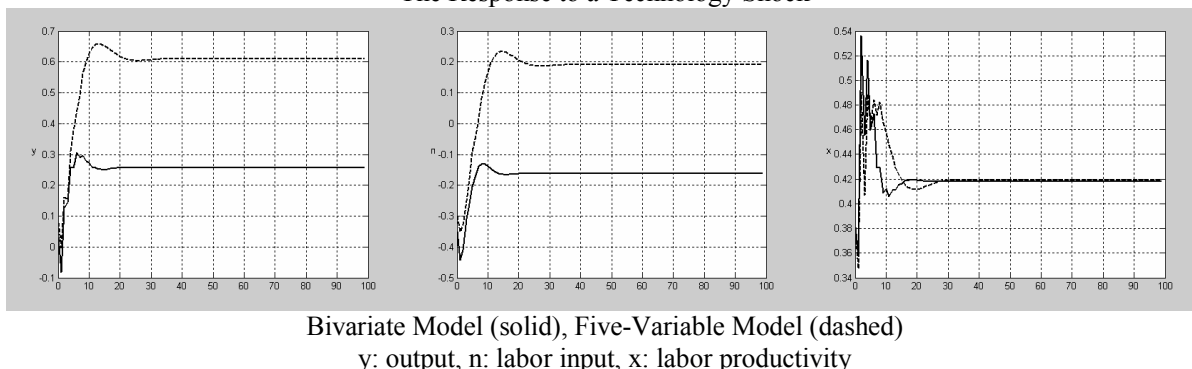


Figure 2: Dynamic Responses in the Bivariate and Five-Variable Gali Models

Table 2: Correlations among Identified Technology Shocks of Bivariate and Augmented Gali Models

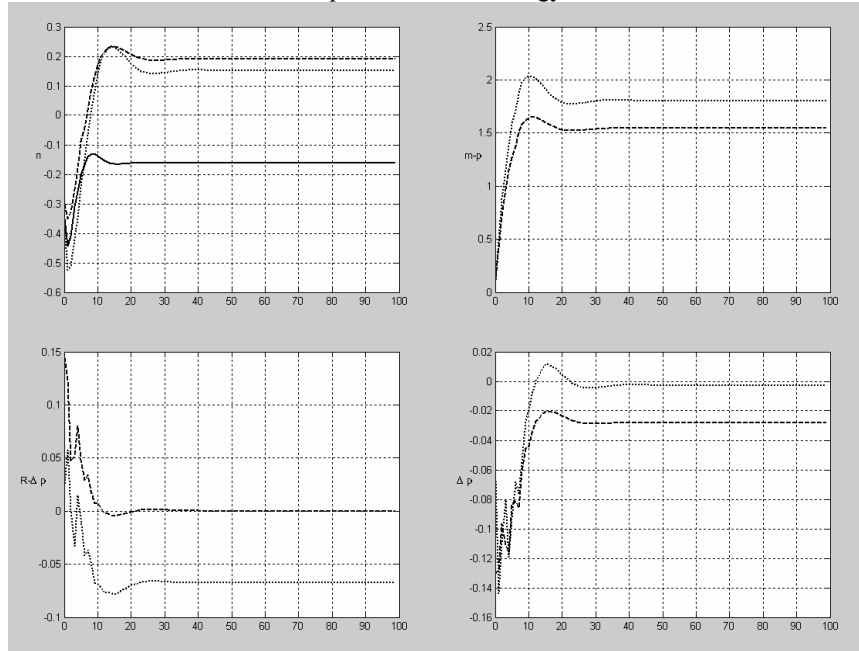
	Gali2V	Gali5V-1	Gali5V-2
Gali2V	1		
Gali5V-1	0.92	1	
Gali5V-2	0.97	0.90	1

Notes: Gali2V refers to the technology shock of the bivariate model. Gali5V-1 stands for the technology shock of the augmented Gali model with not-differenced real interest, Gali5V-2 with differenced real interest.

**The selection of an appropriate variable transformation** In the five-variable model for which some results are reported above, Gali assumes a stationary real interest time series. The stationarity of this variable is, however, disputed<sup>16</sup>. Therefore, the five-variable model is estimated with a first-differenced real interest variable. It is once again observed that the identified technology shocks for all different cases are highly correlated, see Table 2. The dynamic multipliers of output and labor input are affected in comparison to the bivariate model, however only quantitatively. The dynamic responses of labor input, real balances, real interest and inflation are depicted in Figure 3. Whether real interest enters the VAR in level or first-difference plays a role for the quantitative results, but the dynamic responses do not change qualitatively.

<sup>16</sup>The reader can refer to KPSW for a discussion of the stationarity of the real interest series.

### The Response to a Technology Shock



Bivariate Model (solid), 1st Five-Variable Model (dashed), 2nd Five-Variable Model (dotted)  
 n: labor input, m-p: real balances, R- $\Delta p$ : real rate,  $\Delta p$ : inflation

Figure 3: Dynamic Responses to a Technology Shock in the Augmented Gali Models

## 3.2 Blanchard and Quah (1989)

**The selection of the data and the lag order** BQ work with a VAR consisting of the level of unemployment,  $u$ , and the first difference of the logarithm of GNP. Their measure of unemployment is the seasonally adjusted unemployment rate for males, age 20 and over. The reason to use this variable and not the conventional measure of the unemployment rate in the VAR is to discard the effect of demographic changes that possibly exist in the data. The VAR is estimated with eight lags, but this choice is not justified in the paper<sup>17</sup>. BQ only write that estimation with twelve lags produces little difference in the results. The importance of the number of lags in BQ model is investigated below together with the problem of an appropriate variable transformation.

**The selection of an appropriate variable transformation** The specification of the unemployment rate seems especially problematic due to a possible trend in the data as discussed by BQ. Moreover, there are theoretical models which do not exclude that unemployment is a nonstationary variable. BQ nevertheless assume either (trend-) stationarity or structural break in the data and

<sup>17</sup>Different lag selection criteria point to two or three lags.

work with four different data specifications<sup>18</sup>: (a) There is no change in the growth rate of output, but a secular change in the unemployment rate; (b) there is no secular trend in the unemployment rate, but a break in the average growth rate of output; (c) there is neither a change in the growth rate of output nor a secular change in the unemployment rate; (d) there is a break in the growth rate of output as well as a secular trend in the unemployment rate. A fifth specification, (e), is added here where the unemployment rate is assumed to be nonstationary. Specification (e) is perhaps more valid than the first four since for the sample period 1948:1 - 1987:4 the unemployment rate for males over 20 is a nonstationary series according to ADF test results with different test specifications. In other words, even if the secular time trend is removed, what remains is still a nonstationary series according to the test. The last specification implies that supply and demand shocks can have permanent effects on the unemployment rate in contrast to specifications (a) - (d).

(d) is the benchmark model presented in BQ. The findings with respect to the sensitivity to the lag number in the Gali model coincide with the ones from the BQ model. The dynamic multipliers are less sensitive to the lag number in the BQ model, see Figure 4. An observation common to both Gali and BQ models is that the sensitivity of the dynamic multipliers of technology shocks to different lag numbers is higher than the sensitivity for nontechnology shocks as confirmed by Figures 1 and 4.

As mentioned above, BQ estimate with four different specifications of the model variables and report that the results<sup>19</sup> for cases (a)-(c) are qualitatively similar to those of the base case. This is true for almost all the dynamic responses, but the response of unemployment to technology shocks as depicted in Figure 5. In particular, the responses in the first two periods for (a) and (c) are qualitatively different than the responses for (b) and (d) as BQ indicate. Moreover, the impact effect is positive for the latter cases, but not for (a) and (c). Thus, cases (b) and (d) support the finding in Gali (1999) that positive technology shocks lead a decline in the labor input, but not (a) and (c).

Next, the estimated supply and demand components of output and unemployment rate are investigated as Gali (1999) does in Figure 6 of his paper. The output and unemployment business cycles are negatively correlated with a coefficient of -0.90. As illustrated in Figure 6 below, this clear comovement results basically from demand shocks with (d). The demand components of the two

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<sup>18</sup>They argue that there is a break in the mean of output growth starting with 1974:1. They also consider the existence of a secular trend in the unemployment time series.

<sup>19</sup>When BQ write “results”, they probably mean the “impulse response” graphs.

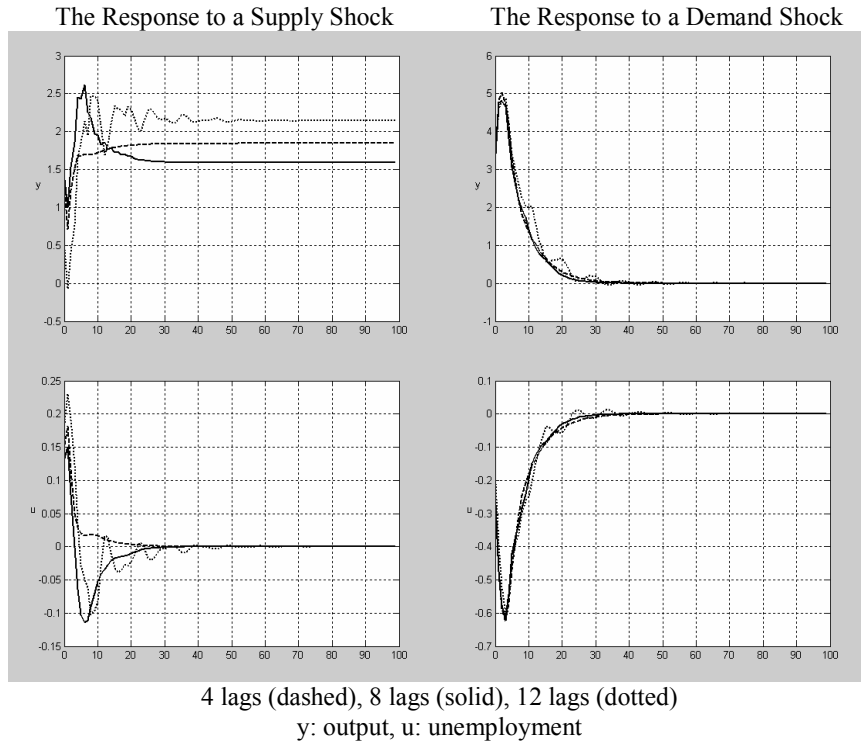


Figure 4: Dynamic Responses to Structural Shocks in the BQ Model

variables are correlated with a coefficient of  $-0.96$  whereas the same coefficient is only  $-0.25$  for the supply components. However, the correlation structure is not clear-cut for the model specifications (a), (c) and (e) as can be seen from Table 3. In particular, the absolute correlation coefficient between the supply components is higher than the one between the demand components with (e).

Table 3 already shows the importance of the way data is treated in the model. In fact, the correlation coefficients among the identified supply and demand shocks, respectively, are all moderately or highly positive. Nevertheless, this is far from being a guarantee for the unimportance of data transformation. The estimated supply and demand shocks exhibit also some relatively high positive or negative correlation, see Table 4. Notice that the correlation coefficient for the supply and demand shocks of the same model is always zero as imposed by the identification scheme. Six of the ten coefficients in the lower triangular part of the table are higher than  $0.50$  in absolute value which points to a lot of noise for the interpretation of the data.

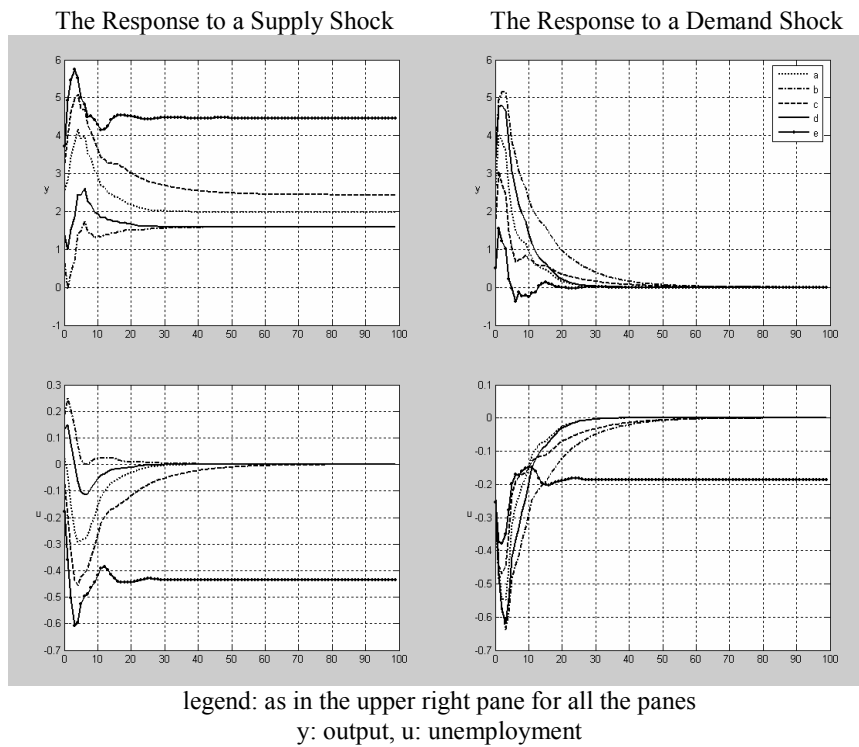


Figure 5: Dynamic Responses with Different Data Specifications in the BQ Model

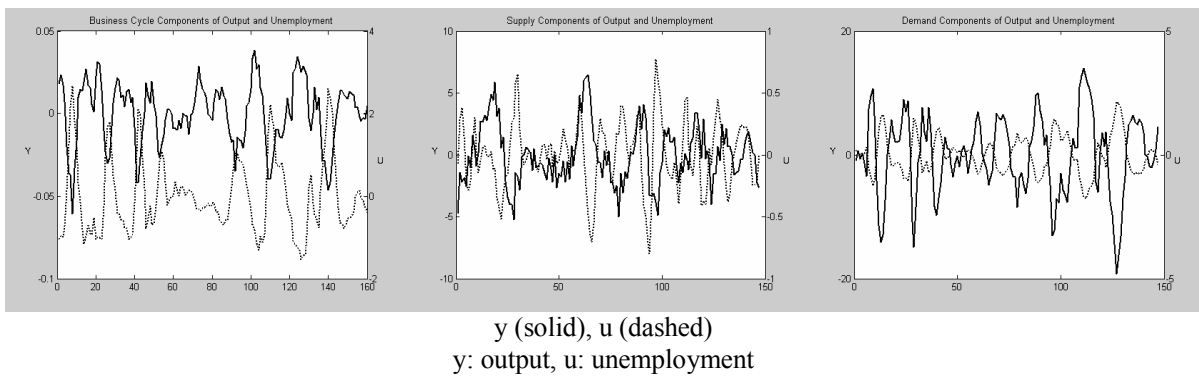


Figure 6: Business Cycle Components of the Variables in the BQ Model

Table 3: Correlations among Supply and Demand Components in BQ Models

	$\text{corr}(y^s, u^s)$	$\text{corr}(y^d, u^d)$
Model (a)	-0.75	-0.98
Model (b)	-0.30	-0.95
Model (c)	-0.88	-0.98
Model (d)	-0.25	-0.96
Model (e)	-0.93	-0.88

Notes:  $x^s$ : The supply component of variable  $x$ .  $x^d$ : The demand component of variable  $x$ .  $y$ : Output.  $u$ : Unemployment rate.

Table 4: Correlations among Estimated Supply and Demand Shocks in BQ Models

	demand <sup>(a)</sup>	demand <sup>(b)</sup>	demand <sup>(c)</sup>	demand <sup>(d)</sup>	demand <sup>(e)</sup>
supply <sup>(a)</sup>	0.00				
supply <sup>(b)</sup>	0.56	0.00			
supply <sup>(c)</sup>	-0.29	-0.77	0.00		
supply <sup>(d)</sup>	0.38	-0.19	0.63	0.00	
supply <sup>(e)</sup>	-0.63	-0.92	-0.37	-0.84	0.00

Notes: supply<sup>(i)</sup> and demand<sup>(i)</sup> refer to the identified supply and demand shocks of model specification ( $i$ ), respectively.

### 3.3 King et al. (1991)

KPSW investigate a three-variable and a six-variable model with long-run restrictions. The difference to the identification schemes above is that the model variables considered in KPSW are cointegrated. The three-variable model consists of output,  $y$ , consumption,  $c$ , and investment,  $i$ . It is augmented later with real balances,  $m - p$ , nominal interest rate,  $R$ , and inflation,  $\Delta p$ . The tests point to a cointegration rank of one in the three-variable model, and three in the six-variable model. A comparison of Figures 2 and 4 in KPSW makes it clear that augmenting the three-variable model leads to important changes in the results with respect to the identified technology shocks.

**Cointegration and identification** KPSW make use of the estimated cointegrating relationships while determining the shape of the  $\hat{A}$  matrix given in equation (5). In Table 2 of their paper, they report the results of three estimated cointegrating relationships in the six-variable system. The first is the money demand equation,

$$m - p = \beta_y y - \beta_R R, \quad (7)$$

and the other two are relationships between the real ratios and the real interest:

$$c - y = \phi_1 (R - \Delta p) \quad (8)$$

and

$$i - y = \phi_2 (R - \Delta p). \quad (9)$$

$\beta_y$ ,  $\beta_R$ ,  $\phi_1$  and  $\phi_2$  are coefficients. The  $A$  matrix in the matrix of long-run multipliers  $\Gamma(1)$  is given by

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & \phi_1 \\ 1 & 0 & \phi_2 \\ \beta_y & -\beta_R & -\beta_R \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \pi_{21} & 1 & 0 \\ \pi_{31} & \pi_{32} & 1 \end{bmatrix}, \quad (10)$$

or

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 + \phi_1\pi_{31} & \phi_1\pi_{32} & \phi_1 \\ 1 + \phi_2\pi_{31} & \phi_2\pi_{32} & \phi_2 \\ \beta_y - \beta_R\pi_{21} - \beta_R\pi_{31} & -\beta_R(1 - \pi_{32}) & -\beta_R \\ \pi_{21} + \pi_{31} & 1 + \pi_{32} & 1 \\ \pi_{21} & 1 & 0 \end{bmatrix}. \quad (11)$$

The order of the variables in the model is  $y, c, i, m - p, R, \Delta p$ . Notice that the  $\hat{A}$  matrix in (10) is written such that the long-run relationships given in equations (7), (8) and (9) hold<sup>20</sup>. Furthermore,  $\hat{A}$  is orthogonal to the cointegrating vectors that would come from (7), (8) and (9)<sup>21</sup>.

KPSW call the first shock a real-balanced-growth shock and indicate that it leads to a unit long-run increase in  $y, c$ , and  $i$ . This is, however, not true as can be detected from (11). The long-run effect of a “balanced-growth” shock is 1,  $1 + \phi_1\pi_{31}$ , and  $1 + \phi_2\pi_{31}$  on  $y, c$ , and  $i$ , respectively<sup>22</sup>. Hence,  $\Delta c - \Delta y = \phi_1\pi_{31}$  in the long run, but not  $\Delta c - \Delta y = 0$ . The second and third permanent shocks are respectively called inflation and real interest shocks by KPSW and they also lead to  $\Delta c - \Delta y \neq 0$ .

Equation (11) tells that output can be affected by only the balanced-growth shock in the long run. Inflation shock is the second permanent shock and it has a unit effect on the inflation rate in the long run. Finally, the reason to call the last shock a “real interest shock” is that it has a unit

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<sup>20</sup>Consider, for example, what happens to (8) in the long-run when a one-time one unit balanced-growth shock occurs. One can express the change in (8) as

$$\Delta c - \Delta y = \phi_1 (\Delta R - \Delta^2 p) \quad (12)$$

with  $\Delta c = 1 + \phi_1\pi_{31}$ ,  $\Delta y = 1$ ,  $\Delta R = \pi_{21} + \pi_{31}$  and  $\Delta^2 p = \pi_{21}$  which follow from equation (11). It is seen that (12) holds when these values are substituted in. This phenomenon is indeed valid for all the cointegrating relationships and all the three permanent shocks.

<sup>21</sup>In particular, one can write

$$\beta = \begin{bmatrix} \beta_y & 0 & 0 & -1 & -\beta_R & 0 \\ 1 & -1 & 0 & 0 & \phi_1 & -\phi_1 \\ 1 & 0 & -1 & 0 & \phi_2 & -\phi_2 \end{bmatrix}. \quad (13)$$

It is easy to verify then that  $\beta\hat{A} = 0$ .

<sup>22</sup>KPSW interpret their identification scheme as if the  $\Pi$  matrix in equation (5) above is an identity matrix which must definitely not be the case.



Table 5: The Estimated Cointegration Equation Parameters in the Six-Variable KPSW Model

Sample	$\beta_y$	$\beta_R$	$\phi_1$	$\phi_2$
1954:1-1988:4	1.197 (1.073;1.321)	0.013 (-0.021;-0.005)	0.0033 (-0.0011;0.0077)	-0.0028 (-0.0128;0.0072)
1949:1-1988:4	1.024	0.010	0.0049	-0.0029
1959:1-1988:4	1.208	0.014	0.0030	-0.0022
1969:1-1988:4	1.126	0.012	0.0030	-0.0025
1974:1-1988:4	1.237	0.012	0.0022	0.0003
1949:1-1973:4	0.514	-0.031	0.0096	-0.0054
1954:1-1973:4	1.244	0.013	0.0018	-0.0081

Notes: The first line contains the 95% confidence intervals of the estimated coefficients in parentheses as reported by KPSW.

impact on the real interest,  $R - \Delta p$ , in the long run<sup>23</sup>.

It is obvious that the estimated coefficients of the cointegration equations may play a role in the KPSW identification approach. In case the coefficients cannot be estimated accurately enough, the identification scheme may lose a lot of explanation power. An investigation of Table 5 shows that KPSW are right in discarding the data before 1954:1 in their six-variable model. The sign of  $\beta_R$ , the coefficient in front of the nominal interest in money-demand equation, in the sample period 1949:1-1973:4 is negative contrary to all the other sample periods.  $\phi_2$  is very close to zero and positive in sample 1974:1-1988:4. The other results are all in line with what is reported and used by KPSW. In particular,  $\beta_R$  seems to be robust to all different sample periods except 1949:1-1973:4.

**The selection of model variables** Contrary to Gali case, augmenting the three-variable model with the nominal variables leads to important changes in the results in KPSW. The differences between the dynamic responses of output, consumption and investment to a balanced-growth shock can be seen by comparing Figures 2 and 4 in KPSW. This section pictures the effects on the identified balanced-growth shock and the business cycle components of the variables with respect to this shock. The estimated correlation coefficients among the balanced-growth shock of the

<sup>23</sup>Alexius and Carlsson (2001) study the six-variable KPSW model and call the three stochastic trends *technology (supply)*, *real interest (demand)*, and *nominal (monetary)*.

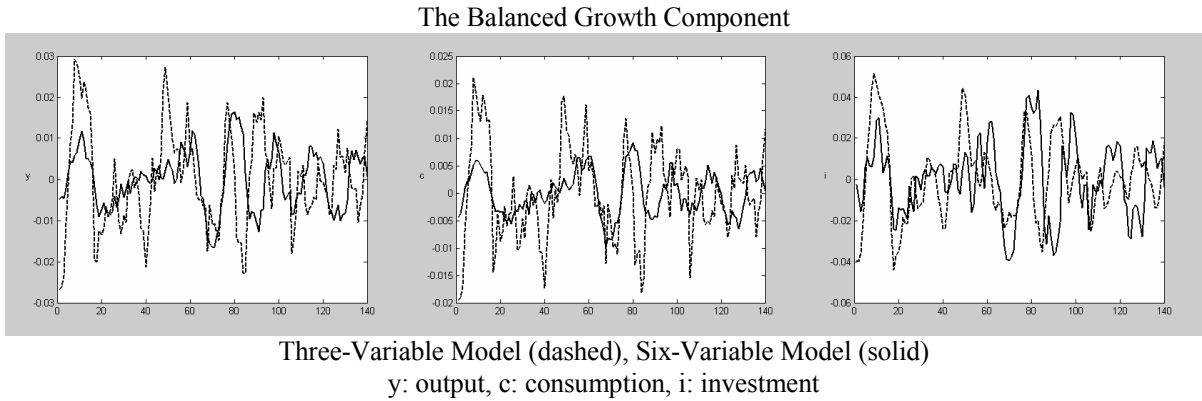


Figure 7: Balanced-Growth Components of Output, Consumption and Investment in the Three- and Six-Variable KPSW Models

three-variable model and the balanced-growth, inflation and real interest shocks of the six variable model are 0.21, -0.17, and 0.55, respectively. The business cycle components of output, consumption, and investment with respect to the balanced-growth shock are illustrated in Figure 7. The corresponding correlation coefficients are only 0.26, 0.37, and 0.25 for output, consumption and investment, respectively. Moreover, the volatility of the output and consumption components is remarkably higher in the three-variable model. These clearly show that the additional variables lead to substantial changes in the results.

Table 6 shows the correlation coefficients for the components of output in three- and six-variable models with respect to the identified shocks. If there are no omitted variable bias and the identification scheme is good enough, it is expected that the balanced-growth output components of both models are very highly correlated and the other cross-correlations are close to zero. Contrary to this expectation, the balanced-growth shocks are only weakly correlated. However, the correlation coefficient among the balanced-growth and real interest components of output of 3- and 6-variable models (0.45) is higher than the correlation coefficient of balanced-growth components (0.26). The same relationship is 0.33 - 0.37 for consumption, and 0.41 - 0.25 for investment.

### 3.4 The Comparison of Different Models

Now follows a comparison across the models. Note that all models contain output<sup>24</sup> and technology/supply/ba growth shocks. Therefore, it is natural to investigate first if the estimated technology shocks and

<sup>24</sup>The Gali models include it indirectly.

Table 6: Correlations among Business Cycle Components in KPSW Models

Sample	$y_{3V}^{bg}$	$y_{6V}^{bg}$	$y_{6V}^{inf}$	$y_{6V}^{ri}$
$y_{3V}^{bg}$	1			
$y_{6V}^{bg}$	0.26	1		
$y_{6V}^{inf}$	-0.01	-0.13	1	
$y_{6V}^{ri}$	0.45	-0.02	0.04	1

Notes: 3V and 6V stand for three- and six-variable models, respectively.  $y_i^j$  is the  $j$  component of the  $i$  model.  $bg$  : Balanced-growth.  $inf$  : Inflation.  $ri$  : Real interest.

the corresponding business cycle components of output are connected to each other. The second important issue is the orthogonality of different structural shocks across the models. If different structural shocks are not approximately orthogonal to each other, this would be a sign for a lot of noise in the data and the researcher should take care when extracting information from the models. Five different models are compared from different viewpoints in this subsection: KPSW3V, KPSW6V, Gali2V, Gali5V, BQ2V<sup>25</sup>. That is, the three- and six-variable KPSW, bivariate and five-variable Gali and BQ models are considered.

Obviously, a coherent data set is needed if one wants to make consistent comparisons. For this reason, the original KPSW data is taken as the underlying data for the common variables<sup>26</sup>. The labor input measure of Gali and the unemployment measure of BQ follow from the original paper and BLS, respectively. The common sample period is chosen as 1954:1-1988:4.

A consistent selection of the lag number seems to be problematic in the literature<sup>27</sup>. As mentioned above, KPSW and BQ use eight lags and Gali four lags in their estimated VAR models. Eight lags are used in all estimations of this section.

**The Comparison across the Basic Models** First, KPSW3V, KPSW6V, Gali2V and BQ2V are compared. It is seen from Table 7 that the structure of the identified technology shocks changes

<sup>25</sup> $iV$  stands for  $i$ -variable.

<sup>26</sup>In particular, output in all the models and the money, price and interest rate in the KPSW and Gali models.

<sup>27</sup>In the studies investigated in this paper, the lag number choice is not justified. KPSW assume eight lags and linear trend in the data when estimating the models, but six lags and quadratic trend when conducting the Johansen cointegration test. However, their results are affected by these choices.

Table 7: Correlations among the Identified Technology Shocks

Model	KPSW3V	KPSW6V	Gali2V	BQ2V
KPSW3V	1			
KPSW6V	0.21	1		
Gali2V	0.14	0.17	1	
BQ2V	0.31	0.14	0.48	1

Notes: Correlations among the identified technology shocks of KPSW3V, KPSW6V, Gali2V and BQ2V models.

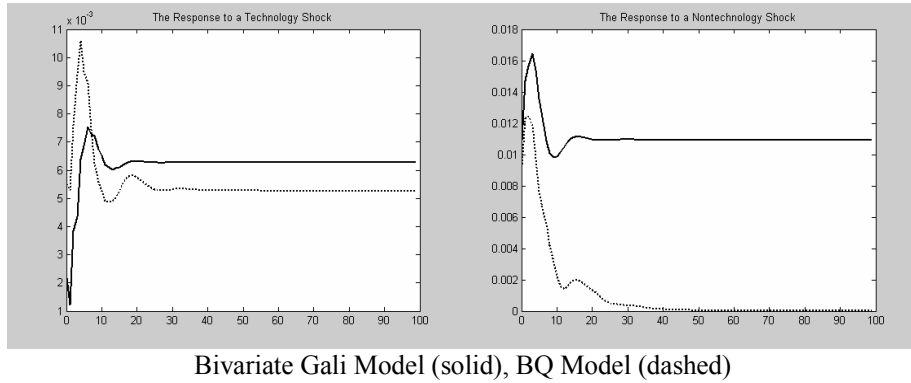


Figure 8: Dynamic Responses of Output in Gali and BQ Models

substantially when the nominal variables are added to KPSW3V: The correlation coefficient among the technology shocks of KPSW3V and KPSW6V is only 0.21. Moreover, both of these technology measures do not seem to be correlated with Gali and BQ measures. On the other hand, the measures of Gali and BQ are moderately correlated with a coefficient of 0.48<sup>28</sup>. This positive coefficient may not be so surprising as similar variables are used in both models. Unemployment rate is a measure that emphasizes the movements in the external margin of the aggregate hours worked. Nevertheless, we should keep in mind an important difference between the Gali and BQ identification schemes: Whereas the Gali scheme allows a long-run impact of demand shocks on output, the BQ scheme does not as reflected in Figure 8. Therefore, the dynamic responses to technology and nontechnology shocks of output indicate partly substantial discrepancies.

Alexius and Carlsson (2005, 2001) and Gali and Rabanal (2004) check if the technology measure

<sup>28</sup>The nontechnology shocks of both models are also correlated with the coefficient of 0.79.

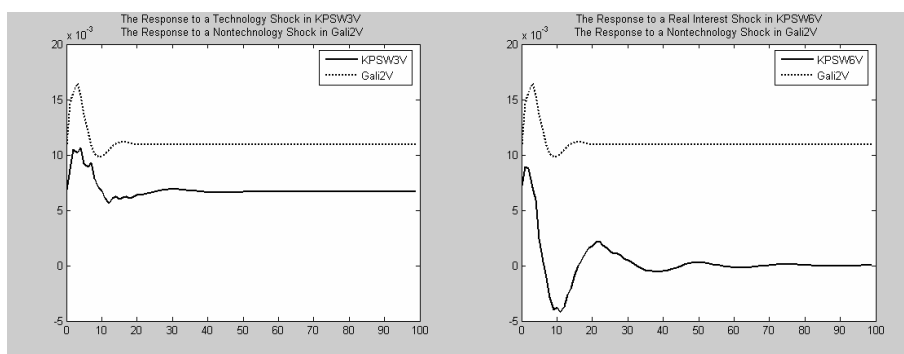


Figure 9: Dynamic Responses of Output

of the Gali model actually captures this phenomenon and come to the conclusion that it does. Therefore, it can be deduced indirectly from Table 7 that the technology measures of both KPSW models do not reflect shocks to technology, a result obtained by Alexius and Carlsson (2005, 2001) before. However, the KPSW3V technology shock is correlated with the Gali2V nontechnology shock with a coefficient of 0.57. The Gali2V nontechnology shock is also correlated with the KPSW6V real interest shock with a coefficient of 0.61. It may therefore be concluded that what is presented as a technology measure in KPSW3V, i.e., the balanced-growth shock, and the real interest shock in KPSW6V seems to *contain substantial nontechnology and/or demand elements*. The illustration of the dynamic responses in Figure 9 endorses this interpretation.

**Gali5V with Cointegration** Gali checks the robustness of his results with an augmented model. It is seen that there are four common variables of KPSW6V and Gali5V: output, real balances, real interest rate and inflation<sup>29</sup>. Although Gali reports no cointegration in his data set, KPSW do. The real ratio relationships do not exist in the Gali model, but at least the money demand relation must exist if KPSW are right. It is unfortunately the case that one cannot easily come to a conclusion about the cointegration rank in KPSW. For some test specifications, it is possible that KPSW6V has a cointegration rank of only two and there is no a priori reason not to assume that these two relationships are the great ratios. Then, it would not make sense to assume a long-run money-demand relationship in Gali5V. But if a cointegration rank of three is assumed in KPSW6V and the third relationship to be the money-demand equation, Gali5V should also have

<sup>29</sup>Clearly, Gali5V does not contain output directly, but indirectly:  $y = x - n$ . Or, KPSW6V contains the nominal interest rate and inflation separately, but not the real interest rate. However, the real interest rate is just a simple combination of the other two variables:  $R - \Delta p$ .

a cointegration rank of at least one.

Two different specifications of Gali5V with cointegration are considered in this paper. The matrix of long-run multipliers in Gali5V1<sup>30</sup> is constructed as:

$$A = \beta_{\perp} \Pi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ * & * & * & * \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ \pi_{21} & 1 & 0 & 0 \\ \pi_{31} & \pi_{32} & 1 & 0 \\ \pi_{41} & \pi_{42} & \pi_{43} & 1 \end{bmatrix}. \quad (14)$$

When the cointegration rank is one and there are five variables in the VAR, four stochastic trends exist and the allowed number of unrestricted parameters in  $\beta_{\perp}$  matrix is one times four, see Subsection 2.2. The identification scheme involves the following properties:

1) The first permanent shock is a technology shock as it is the only shock that has an impact on labor productivity in the long-run. Moreover, this shock is allowed to have an impact on all the variables of the model in the long run as well.

2) The second shock is called a labor supply shock and this is the only shock together with the technology shock that has an impact on labor input in the long-run.

3) The third permanent shock is a real interest shock in the spirit of KPSW which affects the real interest rate by one unit in the long run.

4) The fourth permanent shock is an inflation shock, which can affect only the inflation rate in the long run, but not the real variables except the real balances.

5) The only transitory shock is not given an economic interpretation like in KPSW<sup>31</sup>.

Note that  $\beta_{\perp}$  in Equation (14) should have one free parameter in every column so that the  $\beta' \beta_{\perp} = 0$  condition can be guaranteed. To check the robustness,

$$\beta_{\perp} = \begin{bmatrix} 1 & * & 0 & 0 \\ 1 & 1 & 0 & 0 \\ * & 1 & * & * \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad (15)$$

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<sup>30</sup>Gali5V $j$  stands for the  $j$ -th version of five-variable Gali model with cointegration considered in this study.

<sup>31</sup>This shock may also be given an economic interpretation, but that is not tried in this study as the focus is only on permanent shocks here. The interested reader may refer to Lütkepohl (2006) for how to impose restrictions in order to identify the transitory shocks when there is cointegration.

is also considered in Gali5V2. This new formulation implies that labor productivity is not only affected by technology shocks in the long run, but also by labor supply shocks.  $\beta$  follows from Equation (7) and is written as

$$\beta = \begin{bmatrix} \beta_y & \beta_y & -1 & -\beta_R & -\beta_R \end{bmatrix}. \quad (16)$$

The estimated coefficients  $\beta_y$  and  $\beta_R$  are taken from Table 5.

The augmented Gali model is closely related to the model studied by Shapiro and Watson (1988) and considered also in Alexius and Carlsson (2005). A four-variable VAR with labor input, output, inflation and real rate underlies the study of Shapiro and Watson. They differentiate among three sources, namely labor supply disturbances, technological disturbances, and aggregate demand disturbances that lead to movements in output. They assume that labor input is affected by only technological disturbances in the long-run which is contrary to the identification scheme of Gali5V1 and Gali5V2. Labor productivity is affected by both technological and labor supply supply disturbances like in Gali5V2, but not in Gali5V1. All of the shocks can affect the inflation in the long run. Finally, the augmented Gali model includes real balances in the VAR system whereas the Shapiro-Watson model does not.

**Gali5V without Cointegration** The five-variable Gali model is reinterpreted here. The real interest is assumed to be nonstationary and enters the VAR in first difference<sup>32</sup>. This assumption provides also consistency with KPSW6V. The matrix of long-run multipliers reads

$$C(1) = \begin{bmatrix} * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \\ * & * & * & * & * \end{bmatrix}. \quad (17)$$

The first shock is called technology as it is the only shock with a long-run effect on labor productivity. It is preferred to call the last shock an inflation shock which can affect only the inflation rate in the long run, but not the real variables<sup>33</sup>. Note that the labor productivity is ordered always as the first variable in the VAR, and the inflation rate always as the last variable. This guarantees that regardless of the ordering of labor input, real balances and real rate as the 2nd, 3rd, and 4th

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<sup>32</sup>As shown in Section 3.1 above, how the real interest rate enters the VAR does not affect the results much.

<sup>33</sup>This assumption is does not hold for  $m - p$  in KPSW6V, Gali5V1 and Gali5V2.

variables, the identified technology and inflation shocks and the corresponding dynamic multipliers stay the same<sup>34</sup>. The structural shocks in between are not given an economic interpretation for the time being.

**The Comparison of KPSW6V and Gali5V Models** KPSW6V, Gali5V, Gali5V1 and Gali5V2 contain three, five, four and four permanent shocks, respectively. The identified shocks are

- balanced-growth, inflation and real interest shocks in KPSW;
- technology and inflation shocks in Gali5V; that is, we do not attempt to give an economic interpretation to three of the permanent shocks in this model;
- technology, labor supply, real interest and inflation shocks in Gali5V1 and Gali5V2.

The real interest and inflation shocks and the corresponding dynamic responses are the same in Gali5V1 and Gali5V2. The technology and labor supply shocks of these models are both correlated with a coefficient of 0.98. Hence, the difference in the identification schemes of Gali5V1 and Gali5V2 leads to only minor discrepancies in the results. Given the similarity of these models, Gali5V1 is compared only to KPSW6V and Gali5V in the following.

Table 8 contains the correlation coefficients of the permanent shocks of Gali5V and Gali5V1. The technology and inflation shocks of both models are highly correlated with coefficients of 0.79 and 0.93, respectively. The first uninterpreted shock of Gali5V can be called a labor supply shock as the correlation coefficient of it with the labor supply shock of Gali5V1 is 0.96 and moreover, both of the shocks are approximately orthogonal to all the rest of the shocks. We find it appropriate to call the third uninterpreted shock of Gali5V a real interest shock. For the third uninterpreted shock is correlated with a coefficient of 0.77 with the real interest shock of Gali5V1 and only weakly correlated with the rest. The second uninterpreted shock of Gali5V is correlated with the real interest and inflation shocks of Gali5V1 with coefficients of -0.61 and -0.33, respectively. These numbers lead us to think that this shock could be interpreted as a money supply shock. Nevertheless, it is important to note that the second uninterpreted shock is also moderately correlated with the technology shock of Gali5V1. Figure 10 plots the dynamic responses of output, real balances and real rate in three different panels. The dynamic responses to the third uninterpreted shock in Gali5V and to the real interest shock in Gali5V1 are illustrated in the first column. The dynamic responses to the second uninterpreted shock in Gali5V and to the real interest shock in Gali5V1 are illustrated in the second column. In the third column are plotted the dynamic responses to the second

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<sup>34</sup>See Christiano et al. (1999) for an explanation of this.



Table 8: Correlation among the Identified Structural Shocks of Gali5V and Gali5V1

Model	Gali5V1,tech	Gali5V1,ls	Gali5V1,ri	Gali5V1,inf
Gali5V,tech	0.79	-0.15	0.08	-0.12
Gali5V,1	-0.03	0.96	0.04	-0.06
Gali5V,2	0.44	0.17	-0.61	-0.33
Gali5V,3	0.33	0.13	0.77	-0.04
Gali5V,inf	0.27	0.11	-0.17	0.93

Notes: Correlation among the identified structural shocks of Gali5V and Gali5V1. Gali5V, $i$  stands for the  $i$ th permanent shock in Gali5V which is not given an economic interpretation yet. tech: technology. inf: inflation. ls: labor supply. ri: real interest.

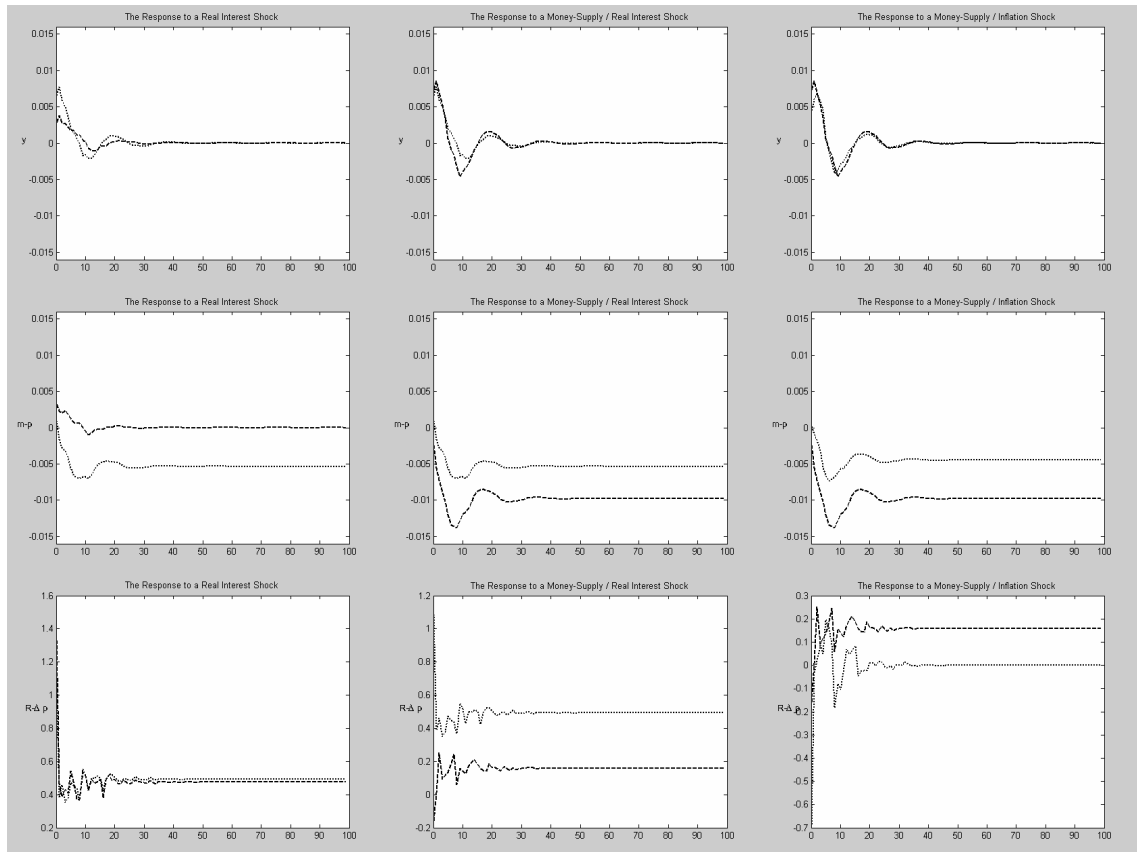
uninterpreted shock in Gali5V and to the inflation shock in Gali5V1. In spite of some discrepancies in these dynamic responses, they have a lot in common both qualitatively and quantitatively. We think that the same phenomenon underlies the money supply, real interest and inflation shocks of both models.

The correlation coefficients reported in Table 9 support our interpretation regarding the monetary, real interest and inflation shocks. The coefficients of the related shocks lie between 0.46 and 0.69 in absolute value. The balanced-growth shock of KPSW6V is weakly correlated with the technology shocks of Gali5V and Gali5V1, but with the labor supply shocks of these models<sup>35</sup>.

Output, real balances and real rate are the common variables of KPSW6V, Gali5V, Gali5V1 and Gali5V2. Figure 11 illustrates the dynamic responses of these variables to technology and inflation shocks. Table 10 reports the correlations among the identified shocks. Both output responses and the dynamic responses of the real balances and real rate to technology shocks resulting from different models correspond to each other. Inflation responses of the real balances and real rate have, however, less common properties.

**Monetary Shocks** It seems so that the money supply, real interest and inflation shocks mentioned above do all correspond to a monetary phenomenon. Therefore, we find it useful to illustrate

<sup>35</sup>Alexius and Carlsson (2005) report a high correlation between the technology shock of KPSW3V and the labor supply shock of Shapiro and Watson (1988).



Gali5V (dashed), Gali5V1 (dotted)  
y: output, m-p: real balances, R-Δp: real rate

Figure 10: Dynamic Responses to Monetary Shocks in the Augmented Gali Models

Table 9: Correlation among the Identified Structural Shocks of KPSW6V, Gali5V and Gali5V1

Model	Gali5V,tech	Gali5V,ls	Gali5V,money	Gali5V,ri	Gali5V,inf
KPSW6V,bg	0.06	0.62	0.37	0.33	0.16
KPSW6V,inf	-0.29	0.11	0.14	-0.31	0.68
KPSW6V,ri	0.15	-0.12	-0.55	0.55	0.44

Model	Gali5V1,tech	Gali5V1,ls	Gali5V1,ri	Gali5V1,inf
KPSW6V,bg	0.35	0.71	0.02	-0.03
KPSW6V,inf	-0.09	0.21	-0.46	0.63
KPSW6V,ri	0.19	-0.11	0.69	0.56

Notes: Correlation among the identified structural shocks of KPSW6V, Gali5V and Gali5V1. bg: balanced-growth. tech: technology. inf: inflation. ls: labor supply. ri: real interest.

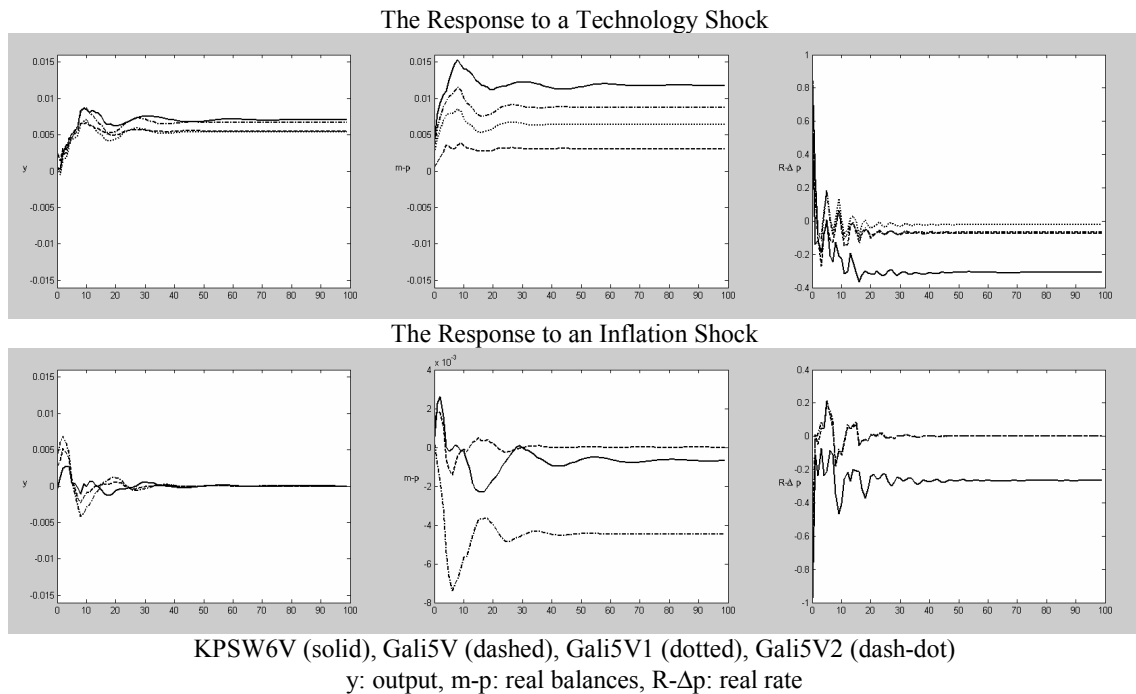


Figure 11: Dynamic Responses to Technology and Inflation Shocks

Table 10: Correlations among the Identified Structural Shocks

Model	KPSW6V	Gali5V	Gali5V1	Gali5V2
KPSW6V	1			
Gali5V	0.06	1		
Gali5V1	0.35	0.79	1	
Gali5V2	0.47	0.75	0.98	1

Technology Shocks

Model	KPSW6V	Gali5V	Gali5V1	Gali5V2
KPSW6V	1			
Gali5V	0.68	1		
Gali5V1	0.63	0.93	1	
Gali5V2	0.63	0.93	1	1

Inflation Shocks

Notes: Correlations among the identified shocks of KPSW6V, Gali5V, Gali5V1 and Gali5V2 models.

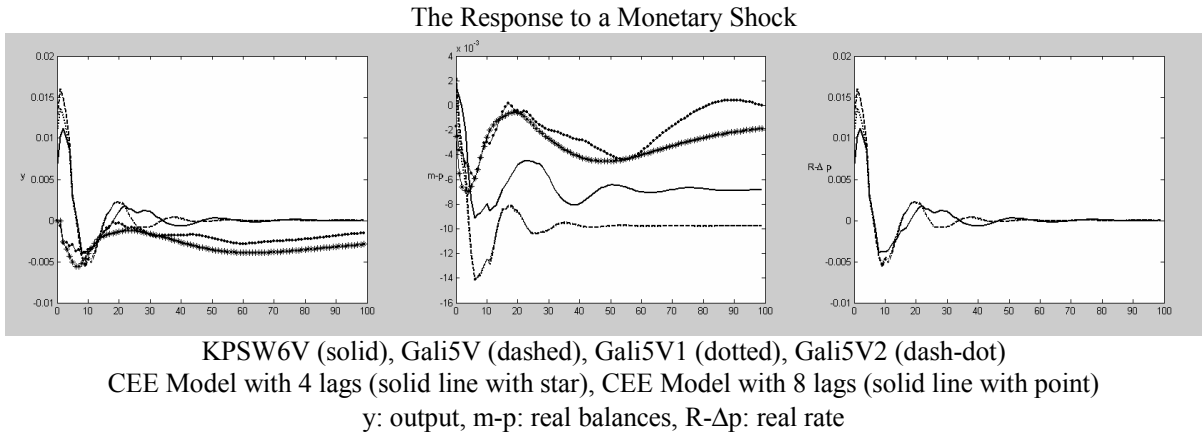


Figure 12: Dynamic Responses to Monetary Shocks

the combined effect of these shocks on the common variables. Moreover, we compare the dynamic responses from our models with dynamic responses coming from the Fed Funds Model used by Christiano et al. (1999)<sup>36</sup>. Note that this is just a naive and preliminary check. The model used by Christiano et al. (1999) is a model that uses short-run restrictions. Furthermore, their model contains different variables than ours<sup>37</sup> and a different sample period. We illustrate the impulse response functions in Figure 12. The responses of the Christiano et al. (CEE) model are computed with four and eight lags. Despite substantial discrepancies, we can observe some similar general tendencies in the dynamic responses of output and real balances. Recall that the impulse response functions of the CEE model are computed with sample period 1966:3 - 1995:2 and the other models with sample period 1954:1 - 1988:4. Thus, this check is just preliminary and not much informative yet. A further comprehensive analysis is the next step in this research project.

### 3.5 Summary and Evaluation

In this section we conducted within-model and across-model comparisons. The within-model comparisons are supposed to show the importance of model specification in SVARs. We first evaluate our findings with respect to model selection decisions.

<sup>36</sup>The dynamic responses for the model where the money measure is M1 are depicted in Figure 2 of this paper.

<sup>37</sup>Namely, log of real GDP, the log of the implicit GDP deflator, the smoothed change in an index of sensitive commodity prices, the federal funds rate, the log of total reserves, the log of nonborrowed reserves plus extended credit, and the log of M2.

**The Choice of the Model Variables** We find that the bivariate and five-variable Gali models arrive at similar results. Although we can extract more information from the five-variable model as shown in Subsection 3.4, the answer to the research question posed by Gali is not affected by augmenting the bivariate model. However, the interpretation in the six-variable KPSW model is very different than the interpretation in the three-variable model. Moreover, the lack of a labor input variable in the KPSW models bias the results substantially.

**The Choice of an Appropriate Variable Transformation** Christiano et al. (2004) oppose the findings of Gali regarding the response of hours to a technology shock: If aggregate hours are assumed to be stationary, that is, if it enters the VAR in level, a technological innovation leads to a rise in hours worked<sup>38</sup>. Yet, the ADF tests point to a unit root for the time series used by Gali.

We investigate what happens if the real rate enters the five-variable Gali VAR in level vs. in first difference in this paper and do not obtain a difference w.r.t. the research question posed. However, the BQ model becomes very noisy when different data specifications are used. We cannot say with any confidence if the business cycle fluctuations are mainly driven by supply or demand shocks based on the different data specifications. The supply and demand shocks of models with different specifications are respectively all positively correlated. But there is also considerable correlation across the identified supply and demand shocks.

**The Selection of the Lag Order** Alexius and Carlsson (2005, 2001) indicate that changing the number of lags has a negligible influence on the identified technology shocks<sup>39</sup>. We obtain the same results for the other structural shocks as well. The dynamic responses are, however, sensitive in some cases both qualitatively and quantitatively. For example, the responses of output and labor input to technology/supply shocks in Gali/BQ models are sensitive to the lag order. On the other hand, there are only marginal discrepancies in responses to nontechnology or demand shocks.

**Cointegration** The restrictions imposed on the cointegrating space makes little difference to resulting technology series in the three-variable KPSW model, see Alexius and Carlsson (2005). We change the orthogonal of the cointegrating space in the five-variable Gali model and observe that the difference is minor in this case as well. The sensitivity of the results in the five-variable Gali model w.r.t. the cointegration rank is also little. KPSW check the sensitivity of their results

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<sup>38</sup>Note also that the labor input measure of Christiano et al. (2004) is different than Gali's measure.

<sup>39</sup>They determine the number of lags by using information criteria and taking care of residual autocorrelation.

w.r.t. the cointegration rank and the cointegrating space and conclude that *the principal results for the base six-variable model are robust to a wide variety of changes in the identifying restrictions.*

Breitung (2000) argues that SVAR methods *should be seen as a more or less useful device to recover structures behind the data. Economic data is used to quantify prior beliefs about the economic system rather than to decide between alternative theories.* Our results show that quantifying prior beliefs with the help of SVAR models is a *dirty business*. The quantitative implications may change significantly with respect to the model selection criteria as seen in some examples in this paper. Faust and Leeper (1997) point to inference problems in SVARs with long-run identification schemes. They indicate that the confidence intervals are not valid with long-run restrictions. Therefore, we believe that prior beliefs cannot be quantified in a reliable way with long-run identification schemes. However, these schemes can be used to recover the main tendencies underlying the macroeconomic data.

The approach presented in this paper provides one possibility to discover structure in the data. SVAR models with different specifications and different variables are compared and an empirical coherence regarding the question of what type of shocks lead mainly to business cycle fluctuations is obtained. Our findings reinforce the reliability of the findings in Gali and BQ and serve to a better understanding of the findings of KPSW. Yet note that we make use of information that comes from outside the SVAR field in order to have confidence in our conclusions.

The SVAR models studied in this paper support further the finding in Gali (1999) that business cycle fluctuations are mainly driven by nontechnology or demand shocks. KPSW argue with their three- and six-variable models that balanced-growth (technology) or real-interest shocks explain most of the variation in business cycle frequencies. We show that both of these shocks are closely related to the nontechnology (demand) measure of Gali (1999). In this respect, the findings of Gali and KPSW coincide: Nontechnology shocks play a bigger role than the technology shocks in the business cycle horizon.

Lippi and Reichlin (1993) show the problem of nonfundamental representations for SVAR studies. Obviously, there may be many possible representations of data that make sense economically. Lippi and Reichlin (1993) indicate for the BQ model that there are two possible nonfundamental representations of the BQ model (they call them E1 and E2) which produce qualitatively similar impulse-response functions, but differ quantitatively. We believe that E2 is an irrelevant example as the response of output to a supply shock is negative in the first periods after the shock occurs

which does not make sense economically. However, E1 seems to be relevant qualitatively. Furthermore, the demand shocks are the driving force according to E1. Although the quantitative implication changes, the main tendency in the data is preserved. We conclude that the problem of nonfundamental representations is therefore not relevant in the case of BQ.

Giannone and Reichlin (2006) emphasize in a recent paper the importance of checking for the possibility of nonfundamentality in a small VAR system by augmenting it with auxiliary variables. They argue that the identified technology shock in Gali (1999) is nonstructural. Given the findings in Alexius and Carlsson (2005) and Gali and Rabanal (2004), we believe that the interpretation in Gali is at least one very strong way to assess the data.

Blanchard and Quah (1993) admit the problem demonstrated by Lippi and Reichlin (1993). They also show that this criticism applies to cointegrated systems more strongly. *Users of common-trends models need to prove that the disturbances they uncover are the ones originally of interest.* This warning applies to the models in KPSW which contain common trends. We think that the use and non-use of KPSW models are made clear in this paper.

The restrictions that are mentioned in Sections 2 and 3 of this paper provide identification of the shocks only up to a sign. As Breitung (2000) shows in a nice way for a bivariate model, the parameters of the  $C$  matrix presented in Subsection 2.1 are identified only in absolute value. There are namely six possible versions of the  $C$  matrix in a bivariate model that would identify the model. The  $C$  matrix is therefore chosen using a priori views about the effects of the structural shocks in a certain period. We choose in the BQ model, for example, the  $C$  matrix which guarantees that the response of output to a positive supply or demand shock is positive in the impact period. Only one possible  $C$  matrix satisfies this condition and the other candidate  $C$  matrices would imply a negative response of output either to a positive supply or to a positive demand shock (or to both) in the impact period. Hence, the other possibilities are discarded. The same applies to the bivariate Gali model, too.

## 4 Conclusion

This paper shows from a different perspective that SVAR with long-run restrictions can be a useful device for empirical analysis. Nevertheless, an SVAR model alone cannot be helpful or reliable as an empirical method. It is necessary to check the ability of an SVAR to capture the phenomenon that it is supposed to capture using other methods than SVAR. SVARs should be used to recover the general tendencies and structures in economic data. In this sense, macroeconomic theories can



be judged with the help of SVAR models. However, quantitative results following from SVARs are rather fragile with respect to model specification.

Nontechnology shocks are the main driving force behind the business cycle fluctuations. In particular, shocks to nominal/monetary variables are found to be important. Supply shocks, for example to the labor supply, play a role in the business cycle horizon together with demand shocks, but not the technology shocks.

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